2013-2014

ECE 386 Homework 3

(a) $T(s) = C(sI - A)^{-1} B$

$$(sI - A) = \begin{bmatrix} s-1 & -1 & 0 \\ -1 & s-1 & 0 \\ 0 & 0 & s+5 \end{bmatrix}$$

$$\operatorname{adj}(sI - A) = \begin{bmatrix} \begin{vmatrix} s-1 & 0 \\ 0 & s+5 \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ 0 & s+5 \end{vmatrix} & \begin{vmatrix} -1 & s-1 \\ 0 & 0 \end{vmatrix} \\ -\begin{vmatrix} -1 & 0 \\ 0 & s+5 \end{vmatrix} & \begin{vmatrix} s-1 & 0 \\ 0 & s+5 \end{vmatrix} & -\begin{vmatrix} s-1 & -1 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ s-1 & 0 \end{vmatrix} & -\begin{vmatrix} s-1 & 0 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} s-1 & -1 \\ -1 & s-1 \end{vmatrix} \end{bmatrix}^T$$

$$\operatorname{adj}(sI - A) = \begin{bmatrix} (s-1)(s+5) & s+5 & 0 \\ s+5 & (s-1)(s+5) & 0 \\ 0 & 0 & (s-1)^2-1 \end{bmatrix} \quad s$$

$$\det(sI - A) = \left[(s-1)^2 - (-1)(-1)\right](s+5) = (s^2 - 2s + 1 - 1)(s+5) = s(s-2)(s+5)$$

$$(sI - A)^{-1} = \frac{\operatorname{adj}(sI - A)}{\det(sI - A)}$$

$$T(s) = [1 \;\; 0 \;\; 1] \frac{\begin{bmatrix} (s-1)(s+5) & s+5 & 0 \\ s+5 & (s-1)(s+5) & 0 \\ 0 & 0 & s(s-2) \end{bmatrix}}{s(s-2)(s+5)} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T(s) = \frac{1}{s+5}$$ (Transfer function)

(b) $d(s) = \det(sI - A) = s(s-2)(s+5)$

eigenvalues $= 0, 2, -5$

(c) For BIBO stability all the poles of transfer function should be at open left half plane

$T(s) = \dfrac{1}{s+5}$ (There is only one pole $s = -5$

$s = -5$ (pole) is at open left half plane

Hence system is BIBO stable

(d) For Lyapunov stability all eigenvalues should be at open left half plane or at the imaginary axis

However $s = 2$ (at open right half plane)

Hence system does not support Lyapunov stability.

(e) The degree of transfer function $T(s)$ is 1

(f) Q = Controllability matrix = $[\,B \;\; AB \;\; A^2B\,]$ size(Q) $\Rightarrow Q_{3\times 3}$ (size(Q) is 3)

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -5 & 25 \end{bmatrix} \qquad \text{rank}(Q) = 1 \neq 3$$

This system is not completely controllable

(g) ϕ = Observability matrix = $\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$ size(ϕ) $\Rightarrow \phi_{3\times 3}$ (size(ϕ) is 3)

$$\phi = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -5 \\ 2 & 2 & 25 \end{bmatrix} \qquad \text{rank}(\phi) = 3 = 3$$

completely observable

(h) While designing a state feedback,

$$\bar{A} = A + bK = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [k_1 \;\; k_2 \;\; k_3]$$

$$\bar{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ k_1 & k_2 & k_3-5 \end{bmatrix} \qquad sI - \bar{A} = \begin{bmatrix} s-1 & -1 & 0 \\ -1 & s-1 & 0 \\ -k_1 & -k_2 & s-k_3+5 \end{bmatrix}$$

$$\det(sI-\bar{A}) = (-1)^{1+1}(s-1)\begin{vmatrix} s-1 & 0 \\ -k_2 & s-k_3+5 \end{vmatrix} + (-1)^{2+1}(-1)\begin{vmatrix} -1 & 0 \\ -k_1 & s-k_3+5 \end{vmatrix}$$

$$+ (-1)^{3+1}\, 0 \begin{vmatrix} -1 & s-1 \\ -k_1 & -k_2 \end{vmatrix}$$

$$\det(sI-\bar{A}) = (s-1)(s-1)(s-k_3+5) + (-1)(s-k_3+5) + 0$$
$$= (s-k_3+5)\left[(s-1)^2-1\right] = (s-k_3+5)\, s(s-2)$$

$$\det(sI-\bar{A}) = s(s-2)(s-k_3+5)$$

desired pole locations using state-feedback $p_1 = p_2 = p_3 = -5$

char-equation: $(s+p_1)(s+p_2)(s+p_3) = (s+5)^2$

if state-feedback is successful

$d(s) = \det(sI-\bar{A})$ with suitable selection of k_1, k_2, k_3

$$(s+5)(s+5)(s+5) \overset{?}{=} s(s-2)(s-k_3+5)$$

Not possible

* It is not possible to re-locate the system poles to $p_1 = p_2 = p_3 = -5$ by state-feedback

(i) Luenberger observer : $H = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$

$$\tilde{A} = A + HC$$

$$\tilde{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -5 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} 1+h_1 & 1 & h_1 \\ 1+h_2 & 1 & h_2 \\ h_3 & 0 & h_3-5 \end{bmatrix} \qquad \det(sI - \tilde{A}) = \det \begin{bmatrix} s-(1+h_1) & -1 & -h_1 \\ -(1+h_2) & s-1 & -h_2 \\ -h_3 & 0 & s-(h_3-5) \end{bmatrix}$$

$$= \det(sI - \tilde{A}) = (-1)^{2+1}(-1)\left[-(s-(h_3-5))(1+h_2) - h_2 h_3\right]$$

$$+ (-1)^{2+2}(s-1)\left[[s-(h_3-5)][s-(1+h_1)] - h_1 h_3\right]$$

$$\det(sI - \tilde{A}) = (s-1)\left[s^2 - (1+h_1+h_3-5)s + (h_1h_3 - 5h_1 + h_3 - 5) - h_1h_3\right]$$

$$- (s - (h_3-5))(1+h_2) - h_2h_3$$

$$= (s-1)\left[s^2 - (h_1+h_3-4)s + (h_3 - 5h_1 - 5)\right]$$

$$- s(1+h_2) + h_2h_3 + h_3 - 5h_2 - 5 - h_2h_3$$

$$= s^3 - [1+h_1+h_3-4]s^2 + (h_1+h_3-4+h_3-5h_1-5)s - (h_3-5h_1-5)$$

$$- s(1+h_2) + h_3 - 5h_2 - 5$$

$$= s^3 - [h_1+h_3-3]s^2 + [-4h_1+2h_3-10-h_2]s + h_3 - 5h_2 - 5$$

$$d(s) = (s+5)(s+5)(s+5) = s^3 + \quad 125$$

$$= (s^2+10s+25)(s+5)$$

$$= s^3 + 15s^2 + 75s + 125$$

$$d(s) = \det(sI - \tilde{A}) \longrightarrow$$

$$15 = 3 - (h_1+h_3)$$

$$75 = 2h_3 - 4h_1 - h_2 - 10$$

$$h_3 - 5h_2 - 5 = 125$$

$$\boxed{h_1 + h_3 = -12}$$

$$\boxed{2h_3 - 4h_1 - h_2 = 85}$$

$$\boxed{h_3 - 5h_2 = 130} \longrightarrow$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 \\ -4 & -1 & 2 \\ 0 & -5 & 1 \end{bmatrix}}_{K} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 85 \\ 130 \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = K^{-1} \begin{bmatrix} -12 \\ 85 \\ 130 \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} -13.8966 \\ -25.6207 \\ 1.8966 \end{bmatrix}$$