LILAŞ BELDEK

Q-1

$$G_0(s) = \frac{K(s+2)(s+4)}{(s-2)(s-4)}$$

$$1 + \frac{K(s+2)(s+4)}{(s-2)(s-4)} = 0 \rightarrow \text{closed loop pole equation}$$

starting points: 2, 4 (2)

ending points: −2, −4 (2)

number of branches = max(m, n) = 2 (2)

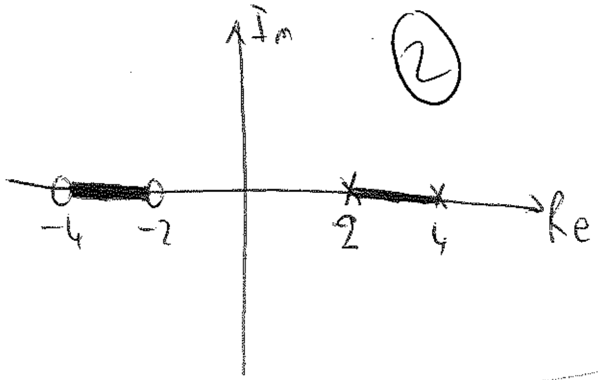
number of asymptotes = |n−m| = 0 (2)

(no asymptote exists hence there is no centroid)

$n = 2 \rightarrow$ # open loop poles

$m = 2 \rightarrow$ # open loop zeros

locus on real axis (2)



breakaway - breakin points

(1) $$\frac{d}{ds}\frac{G_0(s)}{K} = \frac{d}{ds}\frac{(s+2)(s+4)}{(s-2)(s-4)} = \frac{d}{ds}\frac{s^2+6s+8}{s^2-6s+8}$$

$$= \frac{(2s+6)(s^2-6s+8) - (2s-6)(s^2+6s+8)}{(s^2-6s+8)^2}$$

$$= \frac{[2s^3 - 6s^2 - 20s + 48] - [2s^3 + 6s^2 - 20s - 48]}{(s^2-6s+8)^2}$$

$$= \frac{-12s^2 + 96}{(s^2-6s+8)^2} \quad (5)$$

let $-12s^2 + 96 = 0$

$96 = 12s^2 \rightarrow s^2 = 8$

$s = 2\sqrt{2}$ (2)

$$1 + \frac{K(s+2)(s+4)}{(s-2)(s-4)} = 0$$

let $s = 2\sqrt{2}$

$$1 + \frac{K(2\sqrt{2}+2)(2\sqrt{2}+4)}{(2\sqrt{2}-2)(2\sqrt{2}-4)} = 0$$

$$1 + \frac{K(8+8+12\sqrt{2})}{8+8-12\sqrt{2}} = 0 \quad (2)$$

$$1 + \frac{K(16+12\sqrt{2})}{16-12\sqrt{2}} = 0$$

$$K = -\frac{16-12\sqrt{2}}{16+12\sqrt{2}} = \frac{12\sqrt{2}-16}{16+12\sqrt{2}}$$

Q-1 continue

$s = 2\sqrt{2} \Rightarrow K = \dfrac{12\sqrt{2}-16}{12\sqrt{2}+16} = \dfrac{3\sqrt{2}-4}{3\sqrt{2}+4}$ (in root locus since $K>0$)

let $s = -2\sqrt{2}$ $\quad 1 + \dfrac{K(-2\sqrt{2}+2)(-2\sqrt{2}+4)}{(-2\sqrt{2}-2)(-2\sqrt{2}-4)} = 1 + \dfrac{K(8+8-12\sqrt{2})}{8+8+12\sqrt{2}} = 0$

$K = -1 \cdot \dfrac{16+12\sqrt{2}}{16-12\sqrt{2}} = \dfrac{12\sqrt{2}+16}{12\sqrt{2}-16} = \dfrac{3\sqrt{2}+4}{3\sqrt{2}-4}$

2

$s = -2\sqrt{2} \Rightarrow K = \dfrac{3\sqrt{2}+4}{3\sqrt{2}-4}$ (in root locus since $K>0$)

jw axis crossings

$1 + \dfrac{K(s+2)(s+4)}{(s-2)(s-4)} = 0$

$(s-2)(s-4) + K(s+2)(s+4) = 0$

let $s = jw$

$s^2 - 6s + 8 + K(s^2 + 6s + 8) = 0$

5

$-w^2 - 6jw + 8 + K(-w^2 + 6jw + 8) = 0$

$[-w^2 + 8 + K(-w^2 + 8)] = 0 \qquad j[K6w - 6w] = 0$

$[K-1]6w = 0$ 2

either $K = 1$ or $w = 0$

$K = 1 \Rightarrow [-w^2 + 8 + 1(-w^2 + 8)] = 0$ 3

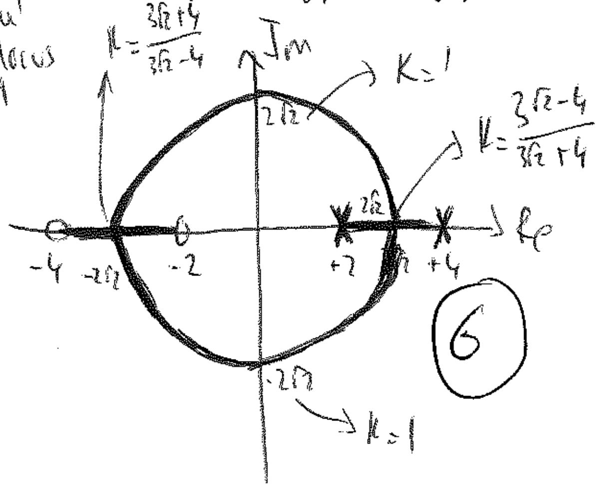
$-2w^2 + 16 = 0 \quad w = \mp 2\sqrt{2}$

in root locus

$w = 0 \Rightarrow [8 + 8K] = 0 \quad K = -1$ (not in root locus)

2

final root locus plot



6

Q-2

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}A & B\\ C & D\end{bmatrix}}_{X}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}K\\ L\end{bmatrix}}_{Y} u \qquad y = \overbrace{\begin{bmatrix}K & L\end{bmatrix}}^{Z}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$H(s) = Z\left[sI - X\right]^{-1} Y$$

$$\left[sI - X\right]^{-1} = \frac{\mathrm{adj}\,(sI - X)}{\det\,(sI - X)}$$

2

$$sI - X = \begin{bmatrix} s-A & -B \\ -C & s-D \end{bmatrix}$$

$$\mathrm{adj}\,(sI - X) = \begin{bmatrix} (s-D) & C \\ B & s-A \end{bmatrix}^T$$

5

$$= \begin{bmatrix} s-D & B \\ C & s-A \end{bmatrix}$$

$$\det\,(sI - X) = (s-A)(s-D) - (-C)(-B)$$

$$= s^2 - (A+D)s + (AD - BC)$$

3

5

$$H(s) = \frac{\begin{bmatrix}K & L\end{bmatrix}\begin{bmatrix} s-D & B \\ C & s-A \end{bmatrix}\begin{bmatrix}K\\ L\end{bmatrix}}{s^2 - (A+D)s + (AD - BC)} = \frac{\begin{bmatrix}K & L\end{bmatrix}\begin{bmatrix} K(s-D) + LB \\ CK + (s-A)L \end{bmatrix}}{s^2 - (A+D)s + (AD - BC)}$$

$$= \frac{K^2(s-D) + KLB + LCK + L^2(s-A)}{s^2 - (A+D)s + (AD - BC)}$$

10

$$= \frac{(K^2 + L^2)s + \left[KLB + LCK - K^2 D - L^2 A\right]}{s^2 - (A+D)s + (AD - BC)}$$

Q-3- $G_0(s) = s^5 - s^4 + 2s^3 + 2s^2 - s + 1$

	$\downarrow$		
s^5	1	2	-1
s^4	-1	2	1
s^3	$c_1=4$	$c_2=0$	0
s^2	$d_1=2$	$d_2=1$	0
s^1	$e_1=-2$	$e_2=0$	0
s^0	$f_1=1$	$f_2=0$	0

14

$$c_1 = \frac{(-1).2 - (1)(2)}{-1} = \frac{-2-2}{-1} = 4$$

$$c_2 = \frac{(-1)(-1) - 1.1}{-1} = \frac{1-1}{-1} = 0$$

$$d_1 = \frac{c_1.2 - (-1)c_2}{c_1} = \frac{4.2 - 0(-1)}{4} = 2$$

$$d_2 = \frac{c_1.1 - (-1).0}{c_1} = \frac{4.1}{4} = 1$$

$$e_1 = \frac{d_1.c_2 - c_1.d_2}{d_1} = \frac{2.0 - 4.1}{2} = -2$$

$e_2 = 0$

$$f_1 = \frac{e_1 d_2 - d_1 e_2}{e_1} = \frac{-2.1 - 2.0}{-2} = 1$$

First column

1
) +
-1
) +
4
) +
2
) +
-2
) +
1

5

There are 4 sign change that means there are 4 open right half plane roots of $G_0(s)$.

Q-4

$$G_0(s) = \frac{4s^3+6s^2+8s+3}{(s^2+1)\ (s^2+2s+2)} =$$

$Y(s) = G_0(s)\,U(s)$ $\quad U(s)=1 \quad Y(s) = G_0(s)$

$\downarrow$ impulse

1

$$Y(s) = \frac{4s^3+6s^2+8s+3}{(s^2+1)\ (s^2+2s+2)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+2}$$

4

$$4s^3+6s^2+8s+3 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+1)$$

5

$$= As^3+(B+2A)s^2+(2B+2A)s+2B+ Cs^3+Ds^2+Cs+D$$
$$= (A+C)s^3+(B+2A+D)s^2+(2B+2A+C)s+(2B+D)$$

$4 = A+C$ $\quad 2B+2A+C=8 \quad 2B+D=3$

$B+2A+D=6$

$A=2 \quad C=2 \quad B=1 \quad D=1$

5

$$Y(s) = \frac{2s+1}{s^2+1} + \frac{2s+1}{s^2+2s+2} = 2\frac{s}{s^2+1} + \frac{1}{s^2+1} + \frac{2s+1+1-1}{(s+1)^2+1^2}$$

2

$$\mathcal{L}^{-1}\Big(Y(s) = 2\frac{s}{s^2+1} + \frac{1}{s^2+1} + 2\frac{(s+1)}{(s+1)^2+1^2} - \frac{1}{(s+1)^2+1^2}$$

2

1

$$y(t) = \left[2\cos(t) + \sin(t) + 2e^{-t}\cos(t) - e^{-t}\sin(t)\right]u(t)$$