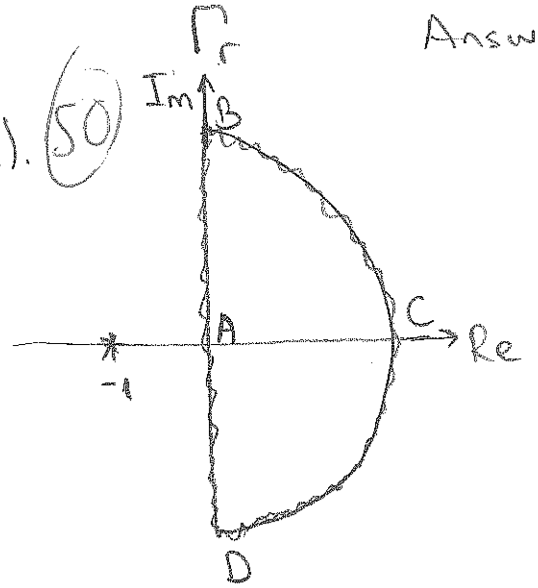
Erhan Işık
200814032

ECE 488 - Final

Answers

Toplam 100

1). 50



$$\Gamma_r = \begin{bmatrix} AB:\ s=jw \quad w: 0 \to \infty \\ BC:\ s=Re^{j\theta} \quad R\to\infty \text{ and } \theta: 90\to 0 \\ CD \text{ is symmetric with } BC \\ DA \text{ is symmetric with } AB \end{bmatrix}$$

In order to find stability check $\frac{H(s)}{K} = \frac{1}{(s+1)^2}$

AB $s=jw$

$$G(s)\Big|_{s=jw} = \frac{H(s)}{K}\Big|_{s=jw} = \frac{1}{(jw+1)^2} = \frac{1}{1-w^2+2jw} = \frac{1-w^2-2jw}{(1-w^2)^2+4w^2}$$

$(1-w^2-2jw)$

$$= \frac{1-w^2}{(1-w^2)+4w^2} + \frac{-2jw}{(1-w^2+4w^2}$$

$$Im\{G(jw)\} = \frac{-2w}{(1-w^2)+4w^2} \qquad Re\{G(jw)\} = \frac{1-w^2}{(1-w^2)+4w^2}$$

	$w=0$	$0<w<1$	$w=1$	$1<w<\infty$	$w=\infty$
$I(G(jw))$	0	$-$	$-\frac{1}{2}$	$-$	0
$Re(G(jw))$	1	$+$	0	$-$	0

BC $s=Re^{j\theta}$

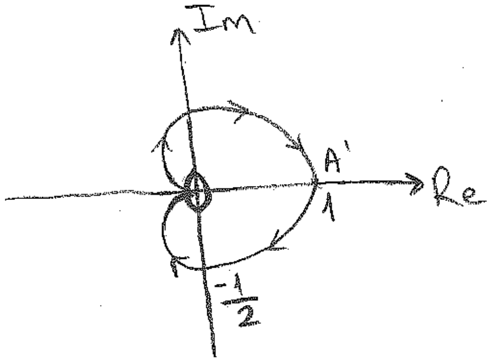
$$G(s)\Big|_{s=Re^{j\theta}} = \frac{H(s)}{K}\Big|_{s=Re^{j\theta}} = \frac{1}{(Re^{j\theta}+1)^2} \cong \frac{1}{R^2e^{2j\theta}} = re^{-2\theta}$$

$\theta=90 \to -2\theta=-180$

$\theta=45 \to -2\theta=-90$

$\theta=0 \to -2\theta=0$

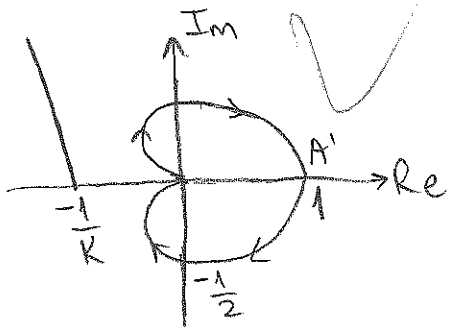
Obtained contour



if $-\frac{1}{K} < 0$

$N = 0$

$N = z - p$

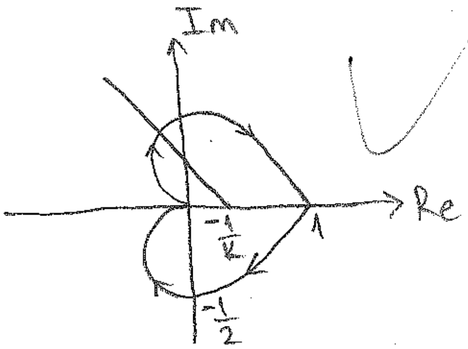


$p = 0$ $\qquad 0 = z - 0$

$z = 0$ stable

$\frac{-1}{K} < 0 \Rightarrow 0 < \frac{1}{K} \Rightarrow K > 0$

if $0 < \frac{-1}{K} < 1$



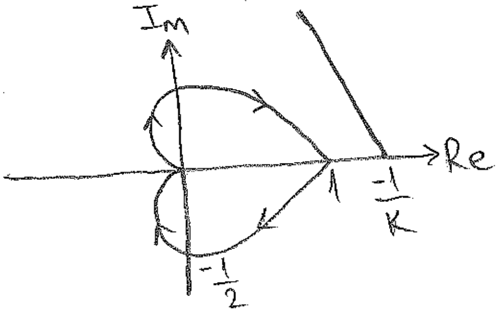
$\frac{-1}{K} < 1 \Rightarrow -1 < \frac{1}{K} \Rightarrow -1 > K$

$N = z - p$

$1 = z - 0 \Rightarrow z = 1$ unstable

if $1 < \frac{-1}{K}$

$\frac{1}{K} < -1 \qquad K > -1$



$N = z - p$

$0 = z - 0$

$z = 0$ stable

<u>Result</u>

if ($K > 0$ or $K > -1$) means that $K > -1$, the system is stable

if $K < -1$, the system is unstable

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2). 35 $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u \qquad y = \underbrace{[1 \quad 0]}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

a). 6 $Q_C = [B \quad AB] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\text{rank}(Q_c) = 2 \quad \det(Q_c) = 1 \neq 0$ the system is completely conrollable

b). 6 $Q_O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\text{rank}(Q_O) = 2 \quad \det(Q_O) = 1 \neq 0$ the systen is completely observable

c). 8 $sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix}$

$\det(sI - A) = s^2 - 1 = (s-1).(s+1)$

$s_1 = 1 \qquad s_2 = -1$

d). 15 $s_{1,2} = -2 \qquad (s+2).(s+2) = s^2 + 4s + 4$

$\det\left(\begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [k_1 \quad k_2]\right)$

$\det\left(\begin{bmatrix} s-k_1 & -1-k_2 \\ -1 & s \end{bmatrix}\right) = s(s-k_1) - (1+k_2) = s^2 - k_1 s - 1 - k_2$

$s^2 - k_1 s - 1 - k_2 = s^2 + 4s + 4$

$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \end{bmatrix} \qquad -k_1 = 4 \qquad -1 - k_2 = 4$

$k_1 = -4 \qquad -k_2 = 5$

$k_2 = -5$

3) 15 $G(s) = \dfrac{\sqrt{2}}{s(s+1)}$

$G(jw) = \dfrac{\sqrt{2}}{jw(jw+1)}$ $\quad |G(jw)| = \dfrac{\sqrt{2}}{w\sqrt{w^2+1}}$ $\quad \angle G(jw) = -90 - \tan^{-1}\dfrac{w}{1}$

$w_c \rightarrow$ gain crossover frequency

$|G(jw_c)| = 1 = \dfrac{\sqrt{2}}{w_c\sqrt{w_c^2+1}}$

$w_c\sqrt{w_c^2+1} = \sqrt{2}$

$w_c^2(w_c^2+1) = 2$ $\quad w_c = a \Rightarrow a(a+1) = 2$

$a^2 + a - 2 = 0$

$(a+2)(a-1) = 0$

$a = -2 \quad a = 1$

$a = w_c^2$

$w_c = \sqrt{1}$

$w_c = 1$

$\phi_m = \angle G(jw)\Big|_{w=w_c} + 180 = -90 - \tan^{-1}\dfrac{1}{1} + 180$

$\phi_m = 90 - 45$

$\boxed{\phi_m = 45}$