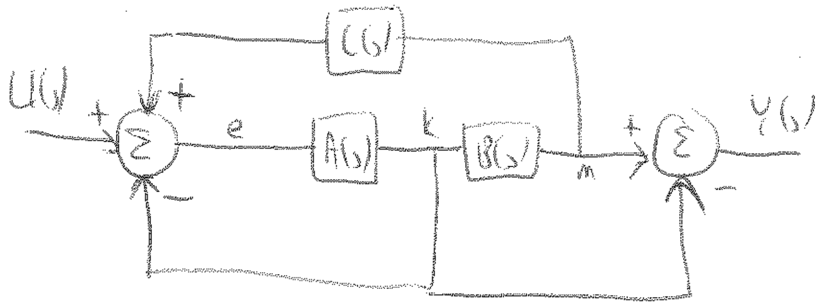
ECE 386 Midterm 20-04-2016

2015-2016 spring

Page 1

Q1



$eA = k \quad kB = m \rightarrow \boxed{eAB = m}$

$u + mC - k = e$

$u + eABC - eA = e \qquad u = e[1 + A - ABC]$

$m - k = Y$

$eAB - eA = Y$

$e[AB - A] = y$

$$\frac{y}{u} = \frac{e[AB - A]}{e[1 + A - ABC]} = \frac{A[B-1]}{1 + A - ABC}$$

(a) $G_o(s) = K \dfrac{(s+2)^2}{(s-1)^2 (s+\cancel{x})}$ breakaway point $s=-3$

Page 2

$= K \dfrac{N(s)}{D(s)}$

$$\left[\frac{d}{ds}(N(s))\right] D(s) - \left[\frac{d}{ds} D(s)\right] N(s) = 0$$

$$2(s+2)(s-1)^2(s+x) - (s+2)^2\left[2(s-1)(s+x) + (s-1)^2\right] = 0$$

$$2(-3+2)(-3-1)^2(-3+x) - (-3+2)^2\left[2(-3-1)(-3+x) + (-3-1)^2\right] = 0$$

$$2(-1)\,16\,(-3+x) - 1\left[2(-4)(-3+x) + 16\right] = 0$$

$$-32(-3+x) = -8\left[-3+x\right] + 16$$

$$96 - 32x = 24 - 8x + 16$$

$$56 = 24x$$

$$x = \frac{7}{3}$$

$G_o(s) = \dfrac{K(s+2)^2}{(s-1)^2 (s+\frac{7}{3})} = K \dfrac{N(s)}{D(s)}$ $\deg(N(s)) = 2 = m$

$\deg(D(s)) = 3 = n$

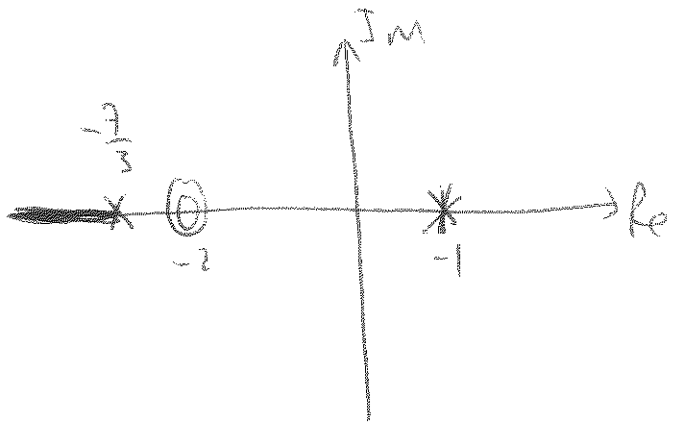
(b) Rule 1 : num_branches = max(m, n) = 3

Rule 2 : Starting points : $p_1 = 1$ $p_2 = 1$ $p_3 = -\frac{7}{3}$

Ending points : $z_1 = -2$ $z_2 = -2$ $z_3 = \infty$ (asymptote)

Page 3

Rule 3: Locus on real axis



Rule 4: Asymptotes: $\theta = \frac{180}{|n-m|} = 180°$

Centroid $= \sigma = \frac{p_1+p_2+p_3-(z_1+z_2)}{|n-m|} = \frac{1+1+\left(-\frac{7}{3}\right)-(-2+-2)}{1}$

$= \frac{6-\frac{7}{3}}{1} = \frac{11}{3}$

Rule 5: Break-in Break-away points

$\left[\frac{d}{ds}N(s)\right]D(s) - \left[\frac{d}{ds}D(s)\right]N(s) = 0$

$2(s+2)(s-1)^2\left(s+\frac{7}{3}\right) - (s+2)^2\left[2(s-1)\left(s+\frac{7}{3}\right) + (s-1)^2\right]$

$(s+2)(s-1)\left[2(s-1)\left(s+\frac{7}{3}\right)\right] - (s+2)(s-1)\left[(s+2)\left[2\left(s+\frac{7}{3}\right)+(s-1)\right]\right]$

$(s+2)(s-1)\left[2s^2+\frac{8}{3}s-\frac{14}{3} - (s+2)\left(3s+\frac{11}{3}\right)\right]$

$(s+2)(s-1)\left[2s^2+\frac{8}{3}s-\frac{14}{3} - 3s^2 - \left(6+\frac{11}{3}\right)s - \frac{22}{3}\right]$

$(s+2)(s-1)\left(-s^2-7s-12\right) = -(s+2)(s-1)(s+3)(s+4)$

$s=-2 \quad s=1 \quad s=-3 \quad s=-4$ break-in, break-away points

Page 4

$$1+G_0(s) = 1+K\frac{(s+2)^2}{(s-1)^2(s+\frac{7}{3})}=0$$

$$(s-1)^2(s+\frac{7}{3})+K(s+2)^2=0$$

if $s=-3$ $\quad (-3-1)^2(-3+\frac{7}{3})+K(-3+2)^2=0$

$$16\left(-\frac{2}{3}\right)+K.1=0 \qquad K=\frac{32}{3}$$

if $s=-4$ $\quad (-4-1)^2(-4+\frac{7}{3})+K(-4+2)^2=0$

$$25\left(\frac{-5}{3}\right)+4K=0 \qquad K=\frac{125}{12}$$

if $K=\frac{125}{12}$ $\quad 1+\frac{125}{12}\frac{(s+2)^2}{(s-1)^2(s+\frac{7}{3})}=0 \Rightarrow s_1=-4,\ s_2=-4 \quad s_3=\frac{-13}{4}$

if $K=\frac{32}{3}$ $\quad 1+\frac{32}{3}\frac{(s+2)^2}{(s-1)^2(s+\frac{7}{3})}=0 \Rightarrow s_1=-3,\ s_2=-3 \quad s_3=-\frac{15}{3}=-5$

Rule 8 → Imaginary axis crossing (should be divisible by w^2+s^2)

$$(s-1)^2(s+\frac{7}{3})+K(s+2)^2=(s^2+w^2)(s+a)$$

$$(s^2-2s+1)(s+\frac{7}{3})+K(s^2+4s+4)=s^3+as^2+w^2s+aw^2$$

$$s^3+\frac{1}{3}s^2-\frac{11}{3}s+\frac{7}{3}+K(s^2+4s+4)=s^3+as^2+w^2s+aw^2$$

$$s^3+\left(\frac{1}{3}+K\right)s^2+\left[4K-\frac{11}{3}\right]s+\left[4K+\frac{7}{3}\right]=s^3+as^2+w^2s+aw^2$$

$a = \frac{1}{3} + K \qquad \omega^2 = 4K - \frac{11}{3} \qquad a\omega^2 = \left(4K + \frac{7}{3}\right)$

$$\left(K + \frac{1}{3}\right)\left(4K - \frac{11}{3}\right) = 4K + \frac{7}{3}$$

$$4K^2 + \left(\frac{4}{3}K - \frac{11}{3}K\right) - \frac{11}{9} = 4K + \frac{7}{3}$$

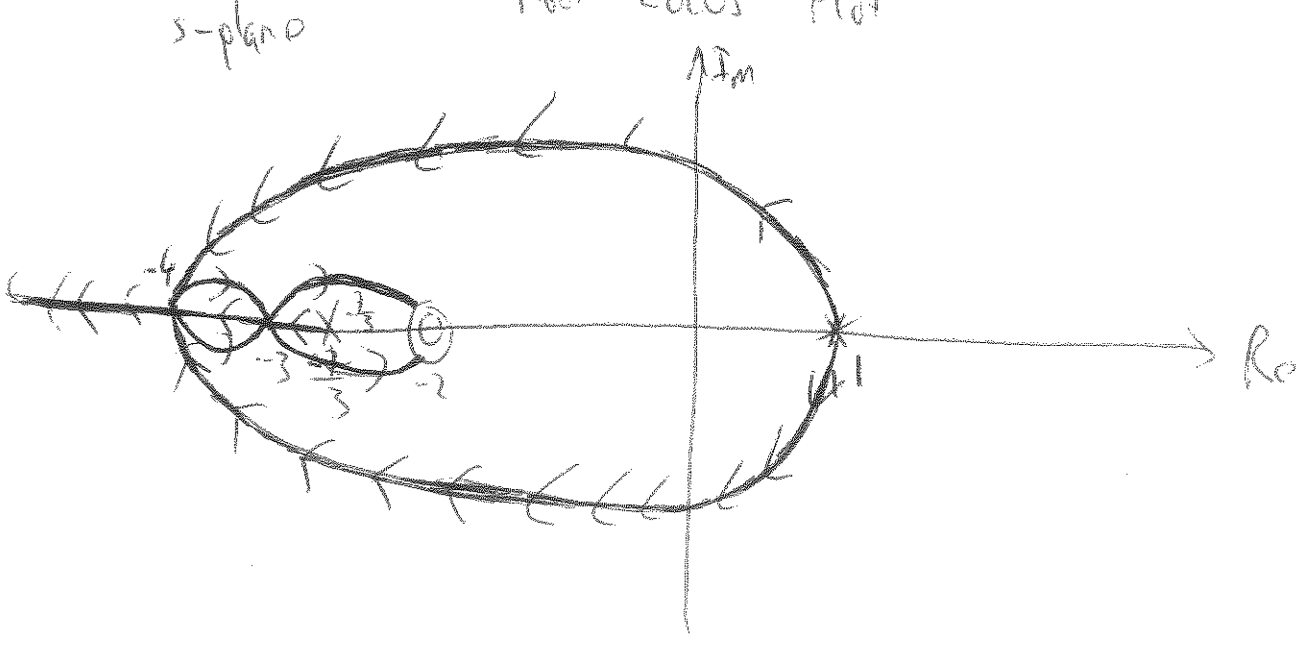
$$4K^2 + \left[-\frac{7}{3} - 4\right]K - \left(\frac{11}{9} - \frac{7}{3}\right) = 0$$

$$4K^2 + \frac{-19}{3}K - \frac{10}{9} = 0$$

$$K = \frac{\frac{19}{3} \pm \sqrt{\frac{361}{9} + \frac{400}{9}}}{8} = \frac{\frac{19}{3} \pm \sqrt{\frac{761}{9}}}{8}$$

take positive $K = \dfrac{\frac{19}{3} + \sqrt{\frac{761}{9}}}{8} = \dfrac{19 + \sqrt{761}}{24}$

Root Locus Plot



Q3 $\dot{x}_1 = x_1^2 + x_2^2 + e^{x_1} - 2 = f_1$ $\qquad y = x_1^2 + x_2^2 = h$

$\dot{x}_2 = x_1 - x_2 + u = f_2$

operating point (set point $y=1$) $[\dot{x}_1 = 0, \dot{x}_2 = 0]$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 + e^{x_1} - 2 \\ x_1 - x_2 + u \end{bmatrix} \qquad \underbrace{x_1^2 + x_2^2} = 1 = y$$

$x_1^2 + x_2^2 + e^{x_1} - 2 = 0 \Rightarrow 1 + e^{x_1} - 2 = 0 \quad e^{x_1} = 1 \quad x_1 = 0$

$x_1 - x_2 + u = 0 \quad 0 - x_2 + u = 0 \quad x_2 = u$

$x_1 = 0 \Rightarrow x_2^2 = 1 = y \quad x_2 = -1$ (it should be non-positive due to the question)

$u = -1$ (it should be non-positive due to the question)

set point $[y = 1, x_1 = 0, x_2 = -1, u = -1]$

$$A = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix}_{\text{set-point}} = \begin{bmatrix} 2x_1 + e^{x_1} & 2x_2 \\ 1 & -1 \end{bmatrix}_{\text{set point}} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{df_1}{du} \\ \frac{df_2}{du} \end{bmatrix}_{\text{set point}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{dh}{dx_1} & \frac{dh}{dx_2} \end{bmatrix}_{\text{set point}} = [2x_1 \quad 2x_2]_{\text{set point}} = [0 \quad -2]$$

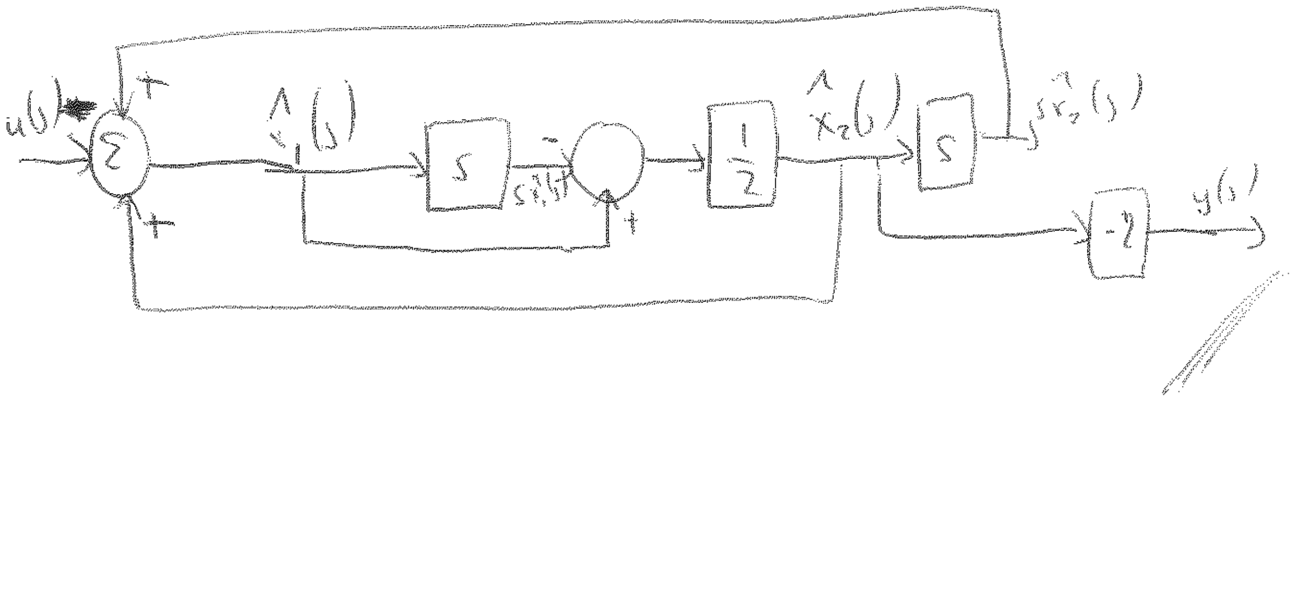
(b) linearized system

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix} = \begin{bmatrix}1 & -2\\ 1 & -1\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}0 & -2\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} \quad \& \quad y(s) = -2\hat{x}_2(s)$$

$$s\hat{x}_1(s) = \hat{x}_1(s) - 2\hat{x}_2(s) \rightarrow \left[\hat{x}_1(s) - s\hat{x}_1(s)\right]\frac{1}{2} = \hat{x}_2(s)$$

$$s\hat{x}_2(s) = \hat{x}_1(s) - \hat{x}_2(s) + u(s) \rightarrow \hat{x}_1(s) = s\hat{x}_2(s) + \hat{x}_2(s) - u(s)$$



Page 8

Q4 $H(s) = \dfrac{1-2s}{1+20s}$

(a) $H(j\omega) = \dfrac{1 - \frac{j\omega}{0.5}}{1 + \frac{j\omega}{0.05}} \Rightarrow |H(j\omega)| = \dfrac{\sqrt{1 + \frac{\omega^2}{0.5^2}}}{\sqrt{1 + \frac{\omega^2}{(0.05)^2}}}$ critical frequencies $\omega = 0.5$ and $\omega = 0.05$

$$|H(j\omega)|_{dB} = 20\log\sqrt{1 + \frac{\omega^2}{(0.5)^2}} - 20\log\sqrt{1 + \frac{\omega^2}{0.05^2}}$$

(1) $0 < \omega < 0.05$

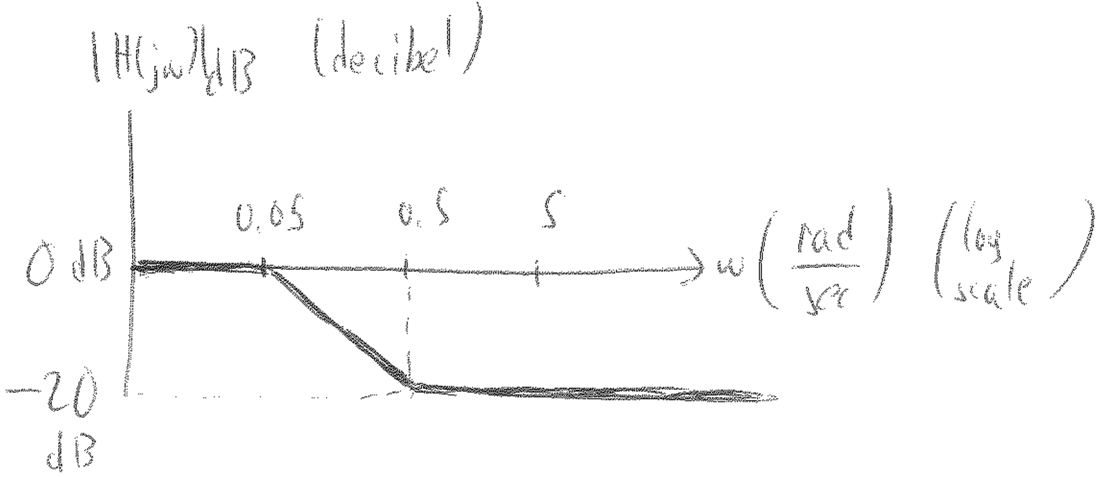
$$|H(j\omega)|_{dB} \simeq 20\log 1 - 20\log 1 = 0\ dB$$

(2) $0.05 < \omega < 0.5$

$$|H(j\omega)|_{dB} \simeq 20\log\sqrt{1} - 20\log\frac{\omega}{0.05}$$

(3) $0.5 < \omega$

$$|H(j\omega)|_{dB} = 20\log\frac{\omega}{0.5} - 20\log\frac{\omega}{0.05} = -20\ dB$$



(b) $$\angle H(j\omega) = \angle \frac{1 - \frac{j\omega}{0.5}}{1 + \frac{j\omega}{0.05}} = + \text{Arctan}\left(\frac{-\omega}{0.5}\right) - \text{Arctan}\left(\frac{\omega}{0.05}\right)$$

$$= -\left[\text{Arctan}\frac{\omega}{0.5} + \text{Arctan}\frac{\omega}{0.05}\right]$$

Critical frequencies — Effect

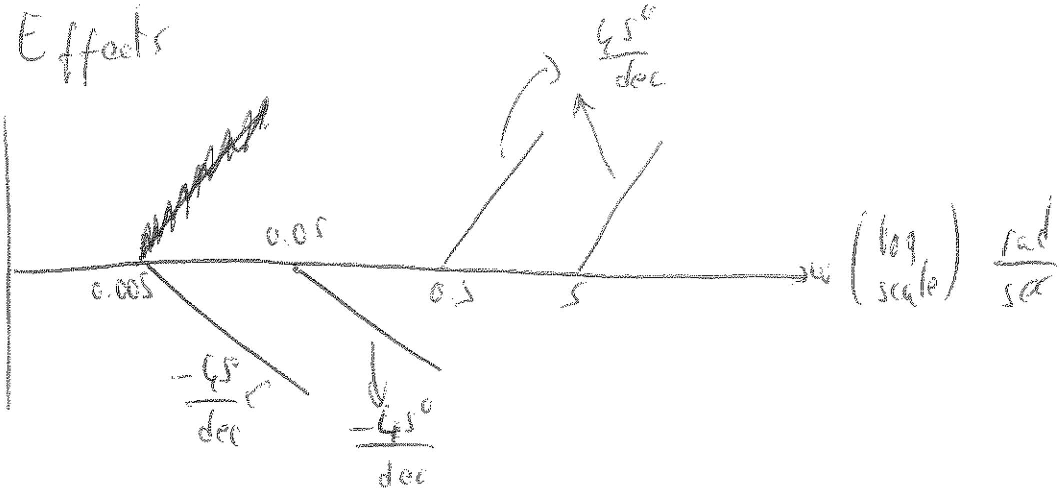
$\omega = 0.05 \rightarrow \frac{\omega}{10} = 0.005 \longrightarrow \frac{-45^\circ}{dec}$

$\omega = 0.05 \rightarrow 10\omega = 0.5 \longrightarrow \frac{45^\circ}{dec}$

$\omega = 0.5 \rightarrow \frac{\omega}{10} = 0.05 \longrightarrow \frac{-45^\circ}{dec}$

$\omega = 0.5 \rightarrow 10\omega = 5 \longrightarrow \frac{45^\circ}{dec}$

Effects



Approximation

— $0 < \omega < 0.005$

$\angle H(j\omega) \simeq 0$

— $0.005 < \omega < 0.05$ total effect $= \frac{-45^\circ}{dec}$

$\angle H(j\omega) \simeq 0^\circ - 45^\circ \log \frac{\omega}{0.005}$ if $\omega = 0.05 \Rightarrow \angle H(j\omega) \simeq -45^\circ$

Page 10

— $0.05 < w < 0.5$ total effect $= \left[-\frac{45^\circ}{dec} + -\frac{45^\circ}{dec} \right] = -\frac{90^\circ}{dec}$

$\angle H(jw) \cong -45 - 90 \log \frac{w}{0.05} \Rightarrow$ if $w = 0.5$ $\angle H(jw) = -135^\circ$

— $0.5 < w < 5$ total effect $= \left(-\frac{45^\circ}{dec} + -\frac{45^\circ}{dec} + \frac{45^\circ}{dec} \right) = -\frac{45^\circ}{dec}$

$\angle H(jw) \cong -135 - 45 \log \frac{w}{0.5} \Rightarrow$ if $w = 5$ $\angle H(jw) = -180^\circ$

— $5 < w$ total effect $= \left(-\frac{45^\circ}{dec} + -\frac{45^\circ}{dec} + \frac{45^\circ}{dec} + \frac{45^\circ}{dec} \right) = \frac{0^\circ}{dec}$

$\angle H(jw) = -180^\circ$

$\angle H(jw)$ (degree)