ECE 386 Homework I

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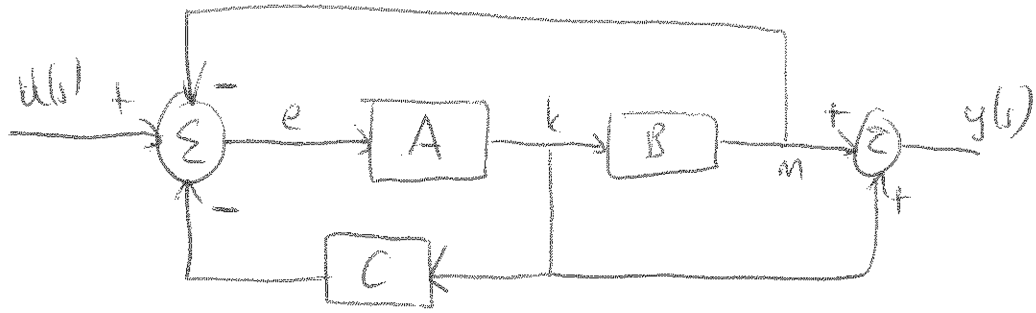
Q1

Ex:



$\boxed{u - m - kC = e}$ $\boxed{eA = k}$ $\boxed{kB = m}$ $\boxed{m + k = y}$

$u - kB - kC = e$ $\quad u - k[B+C] = e$ $\quad u - k[B+C] = \frac{k}{A}$

$u = k\left[B + C + \frac{1}{A}\right]$ $\quad u = k\left[\frac{AB + AC + 1}{A}\right]$

$m + k = y$ $\quad kB + k = y$ $\quad k[B+1] = y$ $\quad k = \frac{y}{B+1}$

$$u = \frac{y}{B+1}\left[\frac{AB + AC + 1}{A}\right]$$

Finally $\quad \frac{y(s)}{u(s)} = \frac{A(B+1)}{AB + AC + 1}$

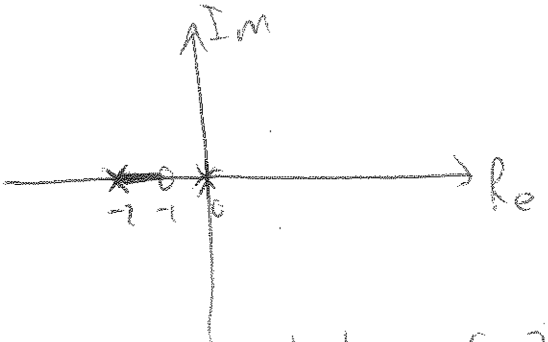
(Q2)

Ex: $G_o(s) = K \dfrac{(s+1)}{s^2\,(s+2)} = \dfrac{N(s)}{D(s)}$ $\quad \deg(N(s)) = 1 = m$
$\quad \deg(D(s)) = 3 = n$

1 – Starting points $\quad p_0 = 0 \quad p_1 = 0 \quad p_2 = -2$
Ending points $\quad z_1 = -1 \quad \boxed{z_2 = \infty \quad z_3 = \infty} \to$ to asymptotes

2 – Num_branches = max(m, n) = max(3, 1) = 3 //

3 – Locus on real axis



4 – Centroid $= \dfrac{p_0 + p_1 + p_2 - [z_1]}{n-m} = \dfrac{0+0+(-2)-(-1)}{2} = -\dfrac{1}{2}$ //

Asymptotes = 90°, −90°

5 – Break-in break-away points

$$\left[\frac{d}{ds} N(s)\right] D(s) - \left[\frac{d}{ds} D(s)\right] N(s) = 0 \Rightarrow \quad s^2(s+2) - \left[3s^2 + 4s\right](s+1) = 0$$

$$s\left[s^2 + 2s\right] - s\left[3s+4\right](s+1) = 0 \qquad s\left[s^2 + 2s - (3s+4)(s+1)\right] = 0$$

$$s\left[s^2 + 2s - 3s^2 - 7s - 4\right] = 0 \qquad s\left[-2s^2 - 5s - 4\right] = 0$$

$-2 \cdot s\left[s^2 + \frac{5}{2}s + 2\right] \longrightarrow$ roots $s_1 = 0$ [starting point is also breakin breakaway points

$$s_{2,3} = \frac{-\frac{5}{2} \mp \sqrt{\frac{25}{4} - 8}}{2} = \frac{-\frac{5}{2} \mp \sqrt{-\frac{7}{4}}}{2} \to \text{complex}$$

(not in root locus $s_{2,3}$)

6- Angle of departure from the poles at the origin (Pg 90)

$$\angle s+1 - 2\angle s - \angle s+2 = 180[2k+1]$$

put $s=0$

$$\angle 1 - 2\,\theta_{departure,\,s=0} - \angle 2 = 180[2k+1]$$

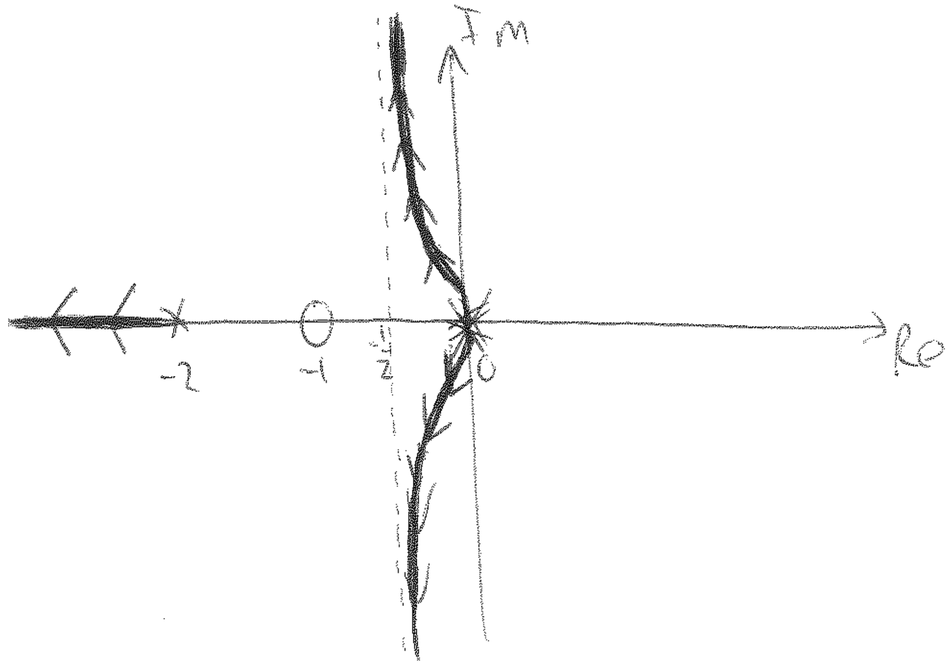
$$0 - 2\,\theta_{departure} = 180[2k+1]$$

$$\theta_{departure} = -90[2k+1]$$

$k=0 \quad \theta_{departure_1} = -90^\circ$

$k=\phi \quad \theta_{departure_2} = 90^\circ$

<u>Total root-locus plot</u>



7 → not available

8 → imaginary axis crossing

$$1+K\,G_0(s) = 1+K\frac{(s+1)}{s^2(s+2)} = 0$$

$$s^2(s+2)+K(s+1) = 0$$

put $s=j\omega$

$$-\omega^2(j\omega+2)+K(j\omega+1)=0$$

$$j\underbrace{\left[-\omega^3+K\omega\right]}_{imaginary}+\underbrace{K-2\omega^2}_{real}=0$$

$\left.\begin{array}{l} imaginary = 0 \\ real = 0 \end{array}\right\}$ together

$\omega=0 \Rightarrow K=0$

(only solution)

Q3 Ex:

$$G_o(s) = \frac{K}{(s+10)^2\,(s+x)} = K\frac{N(s)}{D(s)}$$

(Breakaway, breakin point = -3)

$$\left[\frac{d}{ds}N(s)\right]D(s) - \left[\frac{d}{ds}D(s)\right]N(s) = 0 \quad -\left[2(s+10)(s+x) + (s+10)^2\right] = 0$$

$$-2(s+10)\left[(s+x) + (s+10)\right] = 0$$

$$-2(s+10)\left[2s+10+x\right] = 0$$

$$-4(s+10)\left[s + \frac{10+x}{2}\right] = 0$$

$$s = -10 \quad , \quad s = \frac{-(10+x)}{2}$$

$$-\frac{(10+x)}{2} = -3$$

$$\frac{10+x}{2} = 3 \qquad x = -4$$

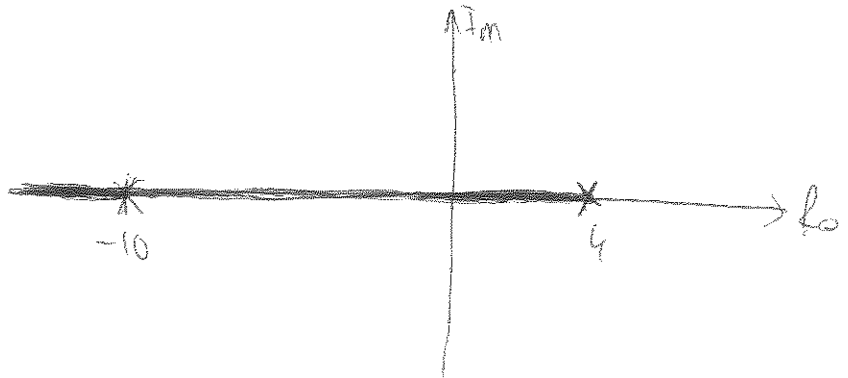
$$G_o(s) = K\frac{1}{(s+10)^2(s-4)}$$

1- Starting points $p_1 = -10 \quad p_2 = -10 \quad p_3 = -4$

Ending points $\boxed{z_1 = \infty \quad z_2 = \infty \quad z_3 = \infty} \rightarrow$ asymptotes

2- Num-branches = max(m,n) = max(3,0) = 3

3- Locus on real axis



4- Centroid $= \dfrac{p_1+p_2+p_3}{(n-m)} = \dfrac{(-10)+(-10)+4}{3} = \dfrac{-16}{3}$

Asymptotes: 60, 180, 300

5- ~~Breakin~~ Breakaway points [found before $s=10$ ~~and~~ $s=-3$]

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$1+G(s)=0$ $\quad 1+\dfrac{K}{(s+10)^2(s-4)}=0$ $\quad$ put $s=-3$

$1+\dfrac{K}{(-3+10)^2(-3-4)}=0$ $\quad 49 \times 7 = K$ $\quad K=343$ positive hence in root locus

6, 7 (not available)

8- Imaginary axis crossing

$1+\dfrac{K}{(s+10)^2(s-4)}=0$

$(s+10)^2(s-4)+K=0$

$[s^2+20s+100][s-4]+K=0$

$s^3+16s^2+20s-400+K=0$

use routh-hurwitz method

s^3	1	20
s^2	16	$K-400$
s^1	$\dfrac{320-K+400}{320}$	0
s^0	$K-400$	0

$\dfrac{720-K}{320}>0$

$K-400>0$

$400<K<720$ for stability

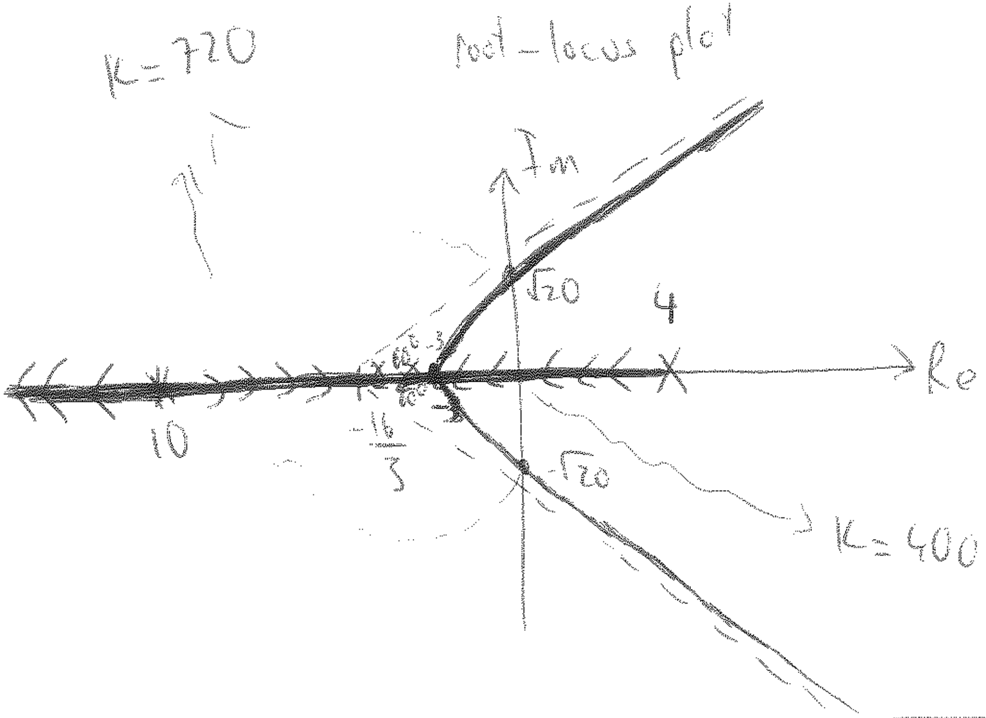
put $s=j\omega$ $\quad -j\omega^3+(-16\omega^2)+20j\omega+(K-400)=0$

$j[20\omega-\omega^3]+[K-400-16\omega^2]$

solution $\quad \omega=0 \Rightarrow K=400$

$\omega=\mp\sqrt{2} \Rightarrow K=720$

root-locus plot



when $K = 400$ $\quad 1+G_0(s)=0 \Rightarrow 1+\frac{400}{(s+10)^2(s-4)}=0$

$$s^3+16s^2+20s=0 \Rightarrow s\,(s^2+16s+20)=0$$

$$s_1=0 \qquad s_{2,3}=\frac{-16\mp\sqrt{256-80}}{2}=-8\mp\frac{\sqrt{176}}{2}=-8\mp\sqrt{44}$$

$$=-8\mp2\sqrt{11}$$

when $K = 720$

$$1+G_0(s)=0 \Rightarrow 1+\frac{720}{(s+10)^2(s-4)}=0 \qquad s^3+16s^2+20s+320=0$$

$$(s^2+20)(s+16)=0$$

$$s_1=-\sqrt{20}\,j \qquad s_2=+\sqrt{20}\,j \qquad s_3=-16$$

Q4 Ex: $p(s) = s^4 + 5s^3 + 3s^2 + 2s - 1$

+	s^4	1	3	-1
+	s^3	5	2	0
+	s^2	$c_1 = \frac{13}{5}$	$c_2 = -1$	$c_3 = 0$
+	s^1	$d_1 = \frac{51}{13}$	$d_2 = 0$	$d_3 = 0$
−	s^0	$e_1 = -1$	0	0

1 sign change

There are "1" roots of $p(s)$ in ORHP

$$c_1 = \frac{5 \times 3 - 2 \times 1}{5} = \frac{15 - 2}{5} = \frac{13}{5}$$

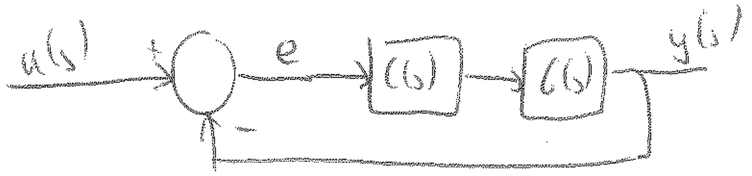
$$c_2 = \frac{5 \times (-1) - 1 \times 0}{5} = -1$$

$$d_1 = \frac{\frac{13}{5} \times 2 - 5 \times (-1)}{\frac{13}{5}} = \frac{\frac{26}{5} + 5}{\frac{13}{5}} = \frac{51}{13}$$

$$d_2 = 0$$

$$e_1 = \frac{\frac{51}{13} \times (-1) - \frac{13}{5} \times 0}{\frac{51}{13}} = -1$$

Q5 Ex:



$$C(s)G(s) = \frac{(s+2)}{s(s+1)(s+5)}$$

$$e(s) = u(s) \frac{1}{1 + C(s)G(s)} \qquad e(s) = \frac{1}{1 + \frac{(s+2)}{s(s+1)(s+5)}} u(s)$$

(a) if $u(s) = \frac{1}{s}$ $\qquad e(s) = \frac{1}{1 + \frac{(s+2)}{s(s+1)(s+5)}} \frac{1}{s}$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\, e(s) = \lim_{s \to 0} \frac{\cancel{s}}{1 + \frac{(s+2)}{s(s+1)(s+5)}} \frac{1}{\cancel{s}}$$

$$= \lim_{s \to 0} \frac{1}{1 + \frac{2}{0 \times 1 \times 5}} = 0$$

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(b) If $u(s) = \frac{1}{s^2}$ $\qquad e(s) = \frac{1}{1 + \frac{s+2}{s(s+1)(s+5)}} \frac{1}{s^2}$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s e(s) = \lim_{s \to 0} \frac{s}{1 + \frac{s+2}{s(s+1)(s+5)}} \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{s(s+1)(s+5)}{s(s+1)(s+5) + (s+2)} \frac{s}{s^2} = \lim_{s \to 0} \frac{(s+1)(s+5)}{s(s+1)(s+5) + (s+2)}$$

$$= \frac{1 \times 5}{0 + 2} = \frac{5}{2}$$

Q6

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Ex

(a)

$$\dot{x}_1 = x_1^2 + x_2^2 + e^{x_1} - 2$$

$$\dot{x}_2 = x_1 - x_2 + u$$

$$y = x_1^2 + x_2^2$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \dot{x} = f(x_1, x_2, u) = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} \qquad y = h(x_1, x_2, u)$$

operating point (set point $y_{sp} = 1$)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{bmatrix} x_1^2 + x_2^2 + e^{x_1} - 2 \\ x_1 - x_2 + u \end{bmatrix} \qquad y = y_{sp} = 1 = x_1^2 + x_2^2$$

$$x_1^2 + x_2^2 = 1 \qquad 1 + e^{x_1} - 2 = 0 \qquad e^{x_1} = 1 \quad x_1 = 0$$

$$0 = x_1 - x_2 + u \qquad x_2 = u$$

$$y_{sp} = 0 \qquad x_{1sp} = 0 \quad \begin{matrix} \nearrow x_{2sp} = 1, -1 \\ \searrow u_{2sp} = 1, -1 \end{matrix}$$

set-points $\boxed{y_{sp} = 1 \quad x_{1sp} = 0 \quad x_{2sp} = 1 \quad u_{2sp} = 1}$ (1)

$\boxed{y_{sp} = 1 \quad x_{1sp} = 0 \quad x_{2sp} = -1 \quad u_{2sp} = -1}$ (2)

linearize at 1

$$A = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix}_{set\text{-}point} \qquad A = \begin{bmatrix} 2x_1 + e^{x_1} & 2x_2 \\ 1 & -1 \end{bmatrix}_{set\text{-}point} = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$

$x_{1sp} = 0$
$x_{2sp} = 1$
$u_{2sp} = 1$
$y_{sp} = 0$

$$B = \begin{bmatrix} \frac{df_1}{du} \\ \frac{df_2}{du} \end{bmatrix}_{set\text{-}point} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{dh}{dx_1} & \frac{dh}{dx_2} \end{bmatrix}_{set\text{-}point} \qquad C = \begin{bmatrix} 2x_1 & 2x_2 \end{bmatrix}_{\substack{x_{1sp}=0 \\ x_{2sp}=1 \\ u_{2sp}=1 \\ y_{sp}=0}} = \begin{bmatrix} 0 & 2 \end{bmatrix}$$

linearized system

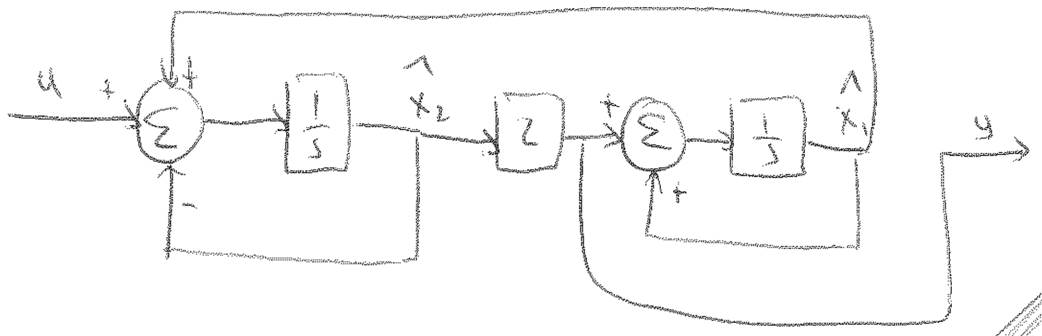
$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0, & 2 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$$

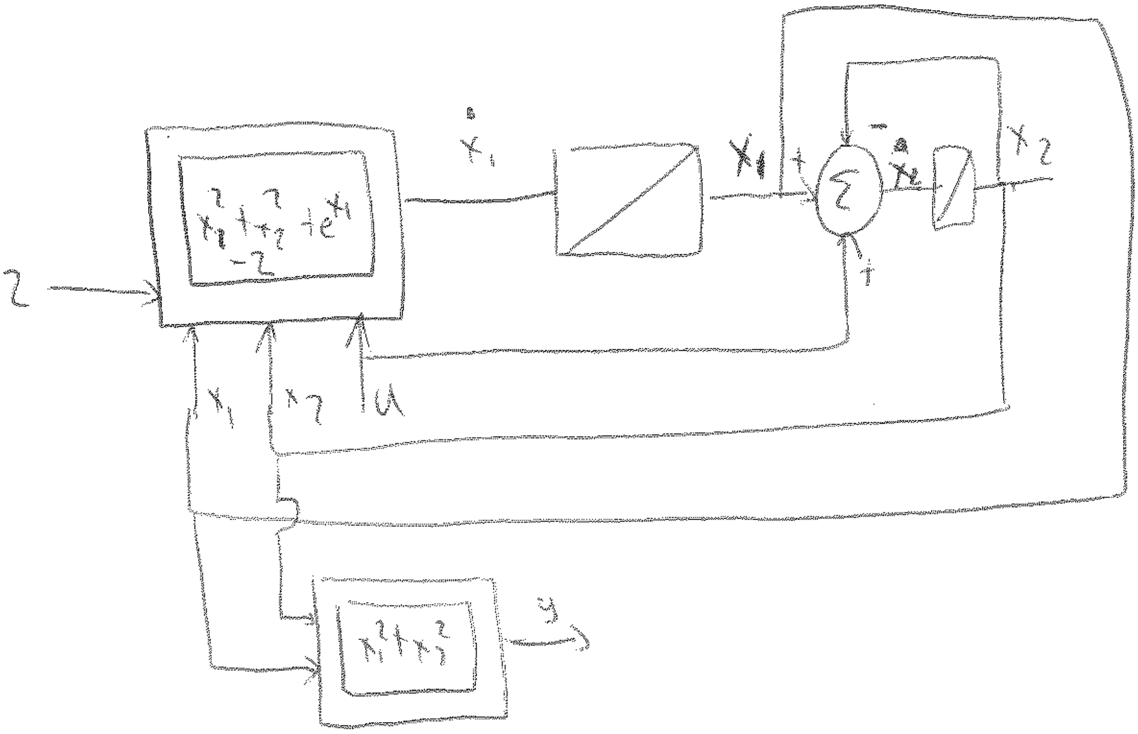
(b) $\dot{\hat{x}}_1 = \hat{x}_1 + 2\hat{x}_2 \xrightarrow{\mathcal{L}} \hat{x}_1(s) = \dfrac{\hat{x}_1(s) + 2x_2(s)}{s}$

$y = 2x_2$

$\dot{\hat{x}}_2 = \hat{x}_1 - \hat{x}_2 + u \xrightarrow{\mathcal{L}} \hat{x}_2(s) = \dfrac{\hat{x}_1(s) - \hat{x}_2(s) + u(s)}{s}$



(c)



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Q7 $H(s) = \dfrac{1-10s}{1-20s}$

a) $H(j\omega) = \dfrac{1-\dfrac{j\omega}{0.1}}{1-\dfrac{j\omega}{0.05}} \rightarrow |H(j\omega)| = \dfrac{\sqrt{1+\dfrac{\omega^2}{0.1^2}}}{\sqrt{1+\dfrac{\omega^2}{0.05^2}}}$ → critical frequencies $\omega = 0.1$ and $\omega = 0.05$

$$|H(j\omega)|_{dB} = 20\log\sqrt{1+\frac{\omega^2}{0.1^2}} - 20\log\sqrt{1+\frac{\omega^2}{0.05^2}}$$

1) $0 < \omega < 0.05$

$|H(j\omega)|_{dB} \cong 20\log 1 - 20\log 1 = 0\ dB$

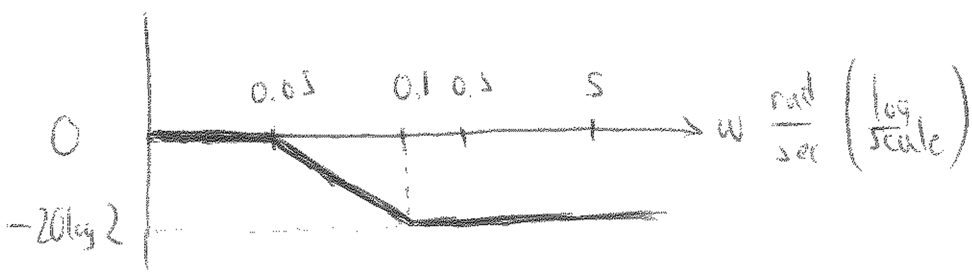
2) $0.05 < \omega < 0.1$

$|H(j\omega)|_{dB} \cong 20\log\sqrt{1} - 20\log\dfrac{\omega}{0.05}$

3) $0.1 < \omega$

$|H(j\omega)|_{dB} \cong 20\log\dfrac{\omega}{0.1} - 20\log\dfrac{\omega}{0.05} = 20\log\dfrac{1}{2} = -20\log 2$

$|H(j\omega)|_{dB}$ (decibel)



(b) $H(jw) = \dfrac{1-\frac{jw}{0.1}}{1-\frac{jw}{0.05}} = \text{Arctan}\left(\dfrac{-w}{0.1}\right) - \text{Arctan}\left(\dfrac{-w}{0.05}\right)$

$= -\text{Arctan}\,\dfrac{w}{0.1} + \text{Arctan}\,\dfrac{w}{0.05}$

Critical frequencies — Effect

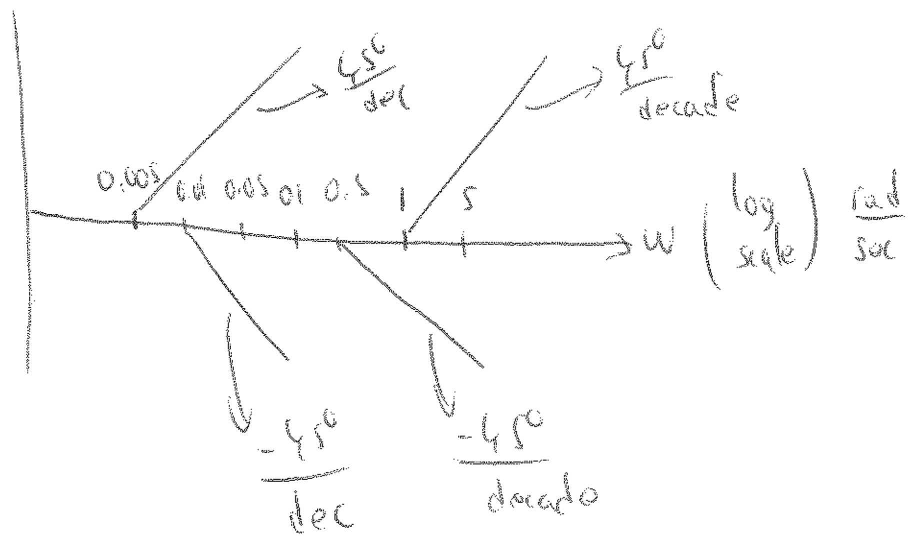
$w = 0.05 \rightarrow \dfrac{w}{10} = 0.005 \longrightarrow \dfrac{45^\circ}{dec}$

$w = 0.05 \rightarrow 10w = 0.5 \longrightarrow \dfrac{-45^\circ}{dec}$

$w = 0.1 \rightarrow \dfrac{w}{10} = 0.01 \longrightarrow \dfrac{-45^\circ}{dec}$

$w = 0.1 \rightarrow 10w = 1 \longrightarrow \dfrac{45^\circ}{dec}$

Effects



Approximation

$0 < w < 0.005$

$\angle H(jw) \cong 0^\circ$

$0.005 < w < 0.01$ (total effect $= \dfrac{45^\circ}{dec}$)

$\angle H(jw) \cong 0^\circ + 45^\circ \log \dfrac{w}{0.005}$ if $w = 0.01 \Rightarrow \angle H(jw) \cong 45 \log 2$

$0.01 < w < 0.5$ (total effect $= \dfrac{45^\circ}{dec} + \left(-\dfrac{45^\circ}{dec}\right) = \dfrac{0^\circ}{dec}$)

$\angle H(jw) \cong 45 \log 2$

$0.5 < \omega < 1$ $\quad$ total effect $= \left(\frac{45^\circ}{dec} + \left(-\frac{45^\circ}{dec} \right) + \left(-\frac{45^\circ}{dec} \right) \right) = -\frac{45^\circ}{dec}$

$\angle H(j\omega) \cong 45 \log 2 - 45 \log \frac{\omega}{0.5}$ $\quad$ if $\omega = 1$ $\quad$ $\angle H(j\omega) = 45 \log 2 - 45 \log 2 = 0^\circ$

$1 < \omega$ $\quad$ total effect $= \left(\frac{45^\circ}{dec} + \left(-\frac{45^\circ}{dec} \right) + \left(-\frac{45^\circ}{dec} \right) + \left(\frac{45^\circ}{dec} \right) \right) = \frac{0^\circ}{dec}$

$\angle H(j\omega) \cong 0^\circ$