ECE 488 – Automatic Control

Feedback Control

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Topics

- Linear system modeling
- Block diagram simplification
- Nonlinear models and set-point linearization
- Stability
- Stead-state and transient response

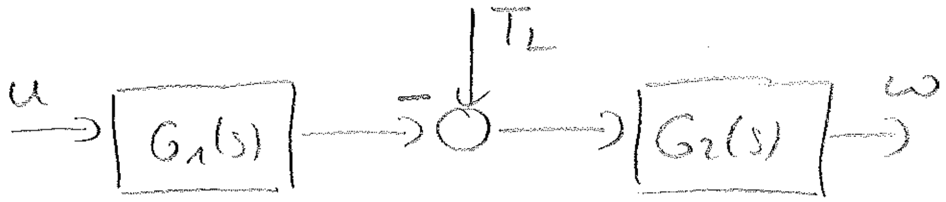
This Week

- Feedback control loop
- Analysis and Properties

Feedback Control Example: DC Motor

Block Diagram

Gap 1



Transfer Functions

$$G_1(s) = \frac{25}{s + 5000}$$

$$G_2(s) = G_d(s) = \frac{32 \cdot 10^4}{(s + 85)}$$

$$G(s) = G_1(s)\, G_2(s) = \frac{8 \cdot 10^6}{(s + 85)\,(s + 5000)}$$

- Keep the speed constant at 20 rad/sec even in case of disturbances

Feedback Control Example: DC Motor

Task

- Keep the rotational velocity constant at 20 rad/sec even in case of disturbances

Possible Strategy

- Proportional correction of output errors

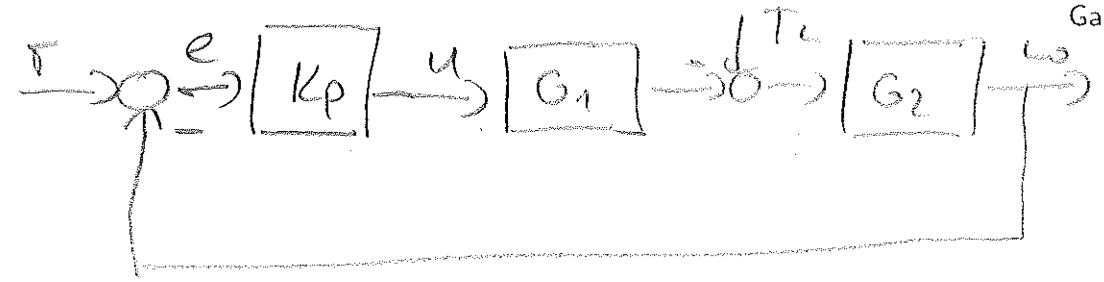
Gap 2

- if speed is too large (error is negative)
  $\Rightarrow$ decrease input voltage
- if speed is too small (error is positive)
  $\Rightarrow$ increase input voltage
- increase / decrease more if error is large

Feedback Control Example: DC Motor

Proportional Control

Gap 3

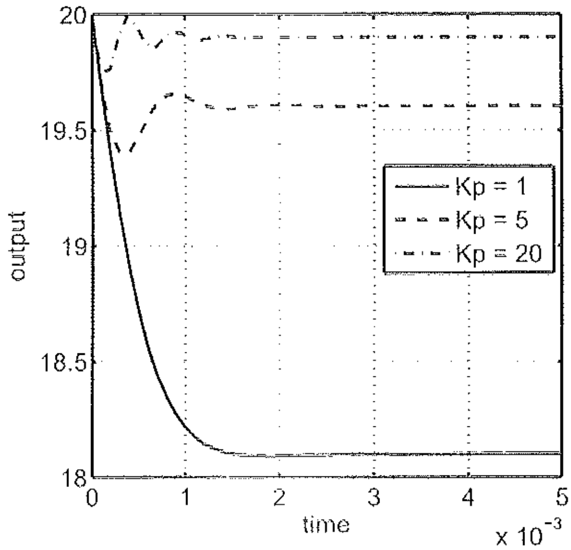


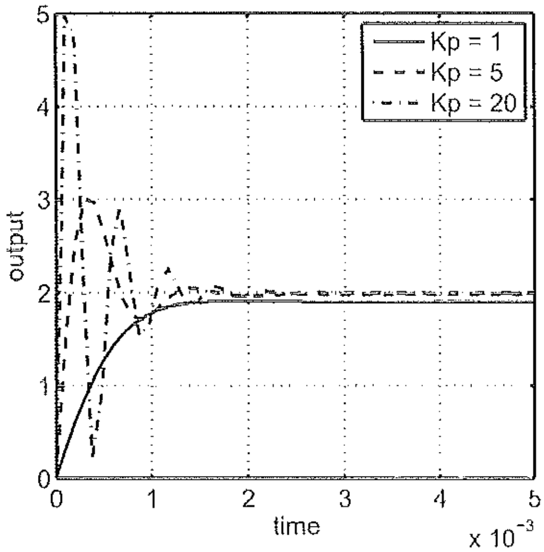
Remarks

- K_p can be considered as controller transfer function
- Different choices of K_p will lead to different behavior of the closed loop
- Larger values of K_p will lead to larger control input signals

Feedback Control Example: DC Motor

Disturbance Step $T_L = 10^{-2}$ Nm





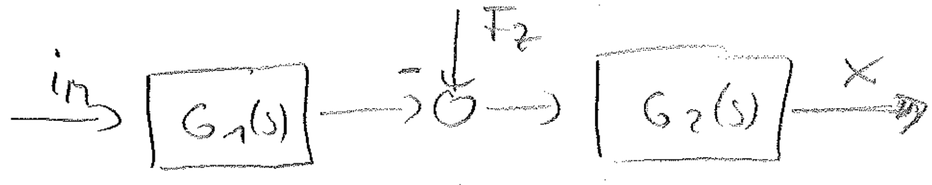
$\Rightarrow$ Stable feedback loop for all choices of K_p

$\Rightarrow$ Steady-state error decreases with larger value of K_p

Feedback Control Example: Magnetic Suspension

Block Diagram

Gap 4



Transfer Functions

$$G_1(s) = 1$$

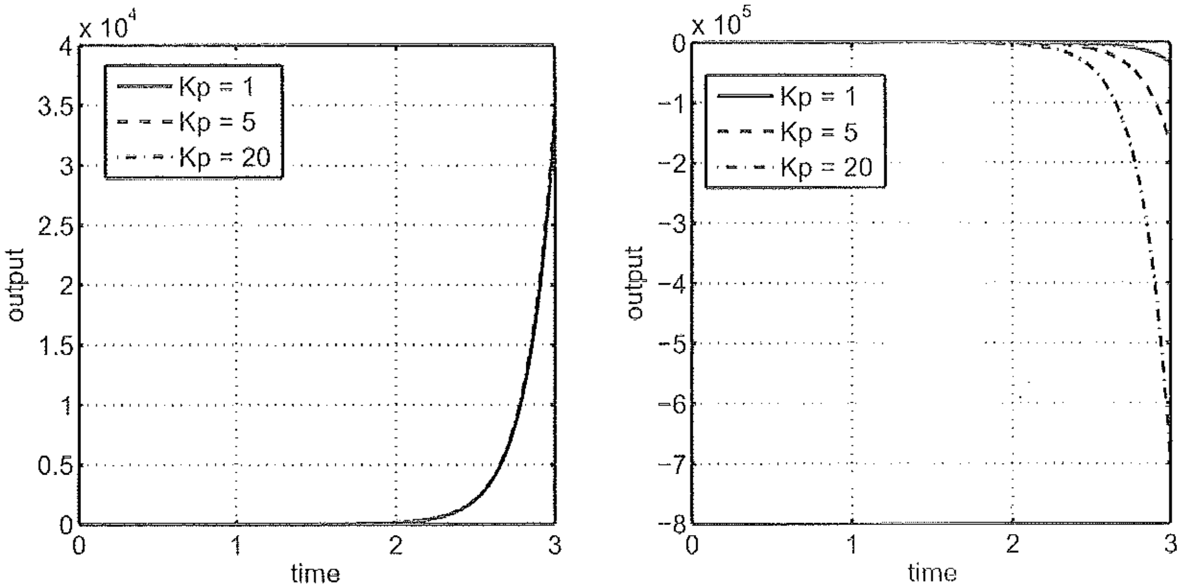
$$G_2(s) = G_d(s) = \frac{0.01}{(-s^2 + 1000)}$$

$$G(s) = G_1(s)\, G_2(s) = \frac{0.01}{(-s^2 + 1000)}$$

- Keep the speed constant at 20rad/sec even in case of disturbances

Feedback Control Example: Magnetic Suspension

Disturbance Step $T_L = 10^{-2}$ N



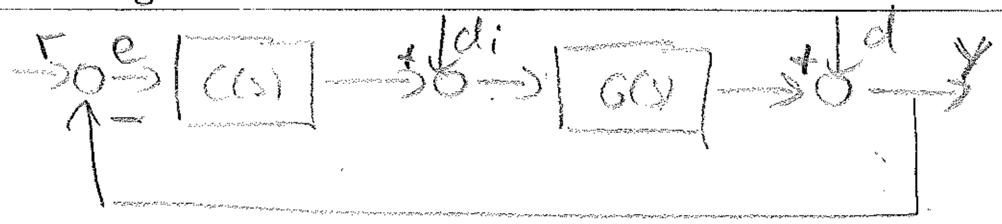
⇒ Instable feedback loop for all choices of K_p

⇒ Proportional control seems not suitable for the magnetic suspension

Basic Feedback Control Loop: Description

Block Diagram

Gap 5



Description

$G(s)$	r	d_i	d
plant transfer function	reference signal	input disturbance signal	output disturbance signal
$C(s)$	e	u	y
controller transfer function	error signal	control input signal	output signal

Basic Feedback Control Loop: Sensitivities

Complementary Sensitivity (Reference signal to output signal)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$$

Sensitivity (Output disturbance signal to output signal)

$$S(s) = \frac{Y(s)}{D(s)} = \frac{1}{1 + C(s)\,G(s)}$$

Input Sensitivity (Input disturbance signal to output signal)

$$S_i(s) = \frac{Y(s)}{D_i(s)} = \frac{G(s)}{1 + C(s)\,G(s)}$$

Control Sensitivity (Reference signal to control input signal)

$$T(s) = \frac{U(s)}{R(s)} = \frac{C(s)}{1 + C(s)\,G(s)}$$

Basic Feedback Control Loop: Sensitivities

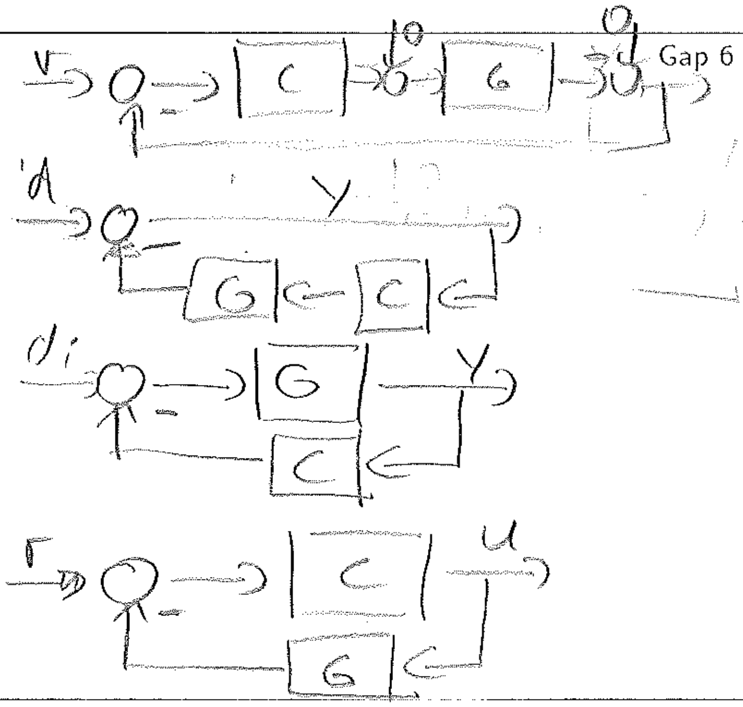
Computation

$$T(s) = \frac{C \cdot G}{1 + C \cdot G}$$

$$S(s) = \frac{1}{1 + C \cdot G}$$

$$S_i(s) = \frac{G}{1 + G \cdot C}$$

$$S_u(s) = \frac{C}{1 + C \cdot G}$$



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Basic Feedback Control Loop: Internal Stability

Definition

The basic feedback loop is **internally stable** *if and only if all four sensitivities are stable transfer functions*

$\Rightarrow$ All signals in the basic feedback control loop are bounded if the input signals r, d, d_i are bounded

Condition

- The feedback loop is internally stable if all zeros of $1 + G(s)C(s)$ lie in the OLHP

General Statements

- If $G(s)$ and $C(s)$ are stable, then stability of $T(s)$ is sufficient for internal stability of the basic feedback control loop
- The feedback loop is instable if $C(s)$ cancels an instable pole or a non-minimum phase zero of $G(s)$ (see next slide and homework)

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Basic Feedback Control Loop: Example

Computation

Gap 7

DC-Motor: $1+C\cdot G = 1+K_p \cdot \frac{8\cdot 10^6}{(s+85)(s+5000)} = 0$

$\Rightarrow s^2+5085s+425000+K_p\cdot 8\cdot 10^6 = 0$

$\Rightarrow$ all coefficients positive ($K_p>0$): stable!

Magnetic Suspension: $1+C\cdot G = 1+\frac{K_p\cdot 0.01}{-s^2+1000} = 0$

$\Rightarrow -s^2+1000+0.01\,K_p = 0$

$\Rightarrow$ coefficient of "s" is always zero

$\Rightarrow$ instable for all values of K_p!

Basic Feedback Control Loop: Instable Pole Cancellation

$C(s)$ must not cancel instable poles of $G(s)$

Gap 8

Assume $G(s)$ has instable pole at $s=p>0$

$\Rightarrow G(s) = \frac{B(s)}{(s-p)\,\hat{A}(s)}$; Then $C(s) = \frac{(s-p)\,\hat{P}(s)}{L(s)}$

cancels the pole. $\Rightarrow S_i(s) = \frac{G}{1+CG} =$

$$= \frac{\frac{B}{(s-p)\hat{A}}}{1+\frac{\cancel{(s-p)}\,\hat{P}\,B}{L\cancel{(s-p)}\,\hat{A}}} = \frac{B\,L}{(s-p)(L\hat{A}+\hat{P}B)}$$

$\Rightarrow S_i$ has instable pole at p

$\Rightarrow$ Feedback loop is not internally stable!

Steady-State Error: Description

Open-loop Transfer Function Type

$$G_o(s) = C(s)\,G(s) = \frac{B(s)}{s^q\,A'(s)}$$

$\Rightarrow$ $G_o(s)$ is of type q

Explanation

- Static position error: Difference between reference and output signal in the steady state for step inputs
- Note: static position error is only bounded if the feedback loop is internally stable

Computation: Final Value Theorem

$$\lim_{t\to\infty}(r(t)-y(t)) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} E(s)\,s = \lim_{s\to 0} S(s)\,\frac{1}{s}\,s = \lim_{s\to 0} S(s)$$

$\Rightarrow$ Static position error is computed from $S(s) = \dfrac{1}{1+C(s)\,G(s)}$

Steady-State Error: Example

Computation

Gap 9

DC-Motor with proportional control

$$S(s) = \frac{1}{1 + K_p \cdot \dfrac{8\cdot 10^6}{(s^2+5085s+425000)}} = \frac{s^2+5085s+425000}{s^2+5085s+425000+K_p\cdot 8\cdot 10^6}$$

$$\Rightarrow \lim_{s\to 0} S(s) = \frac{425000}{425000 + K_p\cdot 8\cdot 10^6}$$

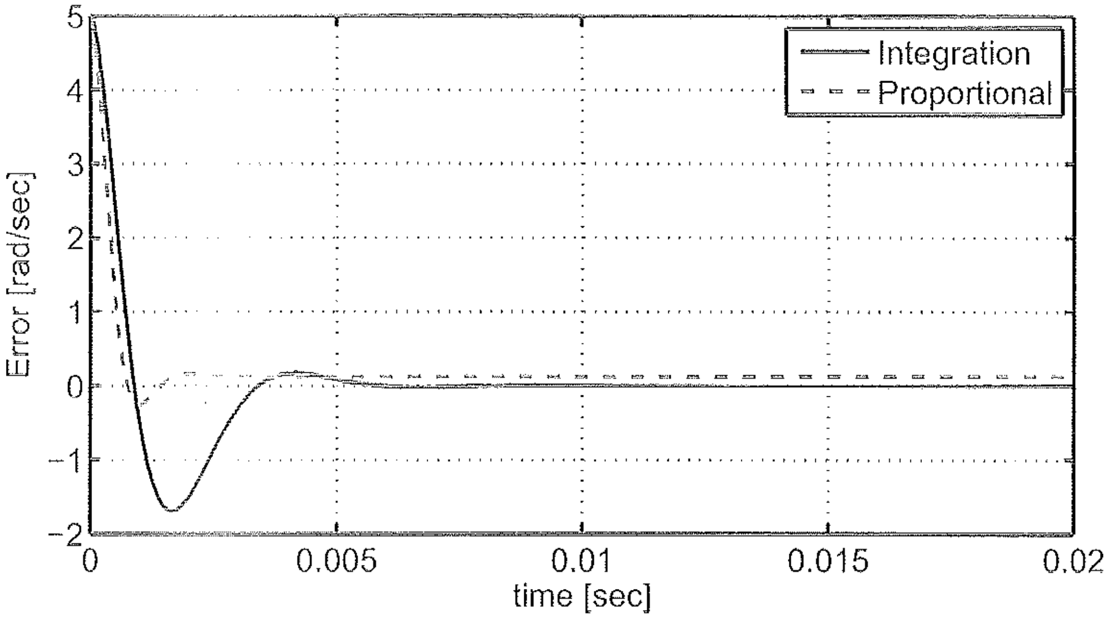
$\Rightarrow$ static position error decreases with increasing K_p

$\Rightarrow$ Proportional control leads to non-zero static position error

$\Rightarrow$ Controller with integrator achieves zero static position error

Steady-State Error: Simulation

DC Motor



Steady-State Error: Velocity and Acceleration

Static Position Error

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} S(s)\,\frac{1}{s^2}\,s = \lim_{s\to 0} S(s)\,\frac{1}{s} = \begin{cases} \text{const} & \text{if } G_o(s) \text{ is type } 0 \\ 0 & \text{if } G_o(s) \text{ is type} > 0 \end{cases}$$

Static Velocity Error

- Difference between reference and output signal in the steady state for ramp inputs

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} S(s)\,\frac{1}{s^2}\,s = \lim_{s\to 0} S(s)\,\frac{1}{s} = \begin{cases} \infty & \text{if } G_o(s) \text{ is type } 0 \\ \text{const} & \text{if } G_o(s) \text{ is type } 1 \\ 0 & \text{if } G_o(s) \text{ is type} > 1 \end{cases}$$

Static Acceleration Error

- Difference between reference and output signal in the steady state for ramparabolic inputs

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} S(s)\,\frac{1}{s^2}\,s = \lim_{s\to 0} S(s)\,\frac{1}{s} = \begin{cases} \infty & \text{if } G_o(s) \text{ is type} < 2 \\ \text{const} & \text{if } G_o(s) \text{ is type } 2 \\ 0 & \text{if } G_o(s) \text{ is type} > 2 \end{cases}$$

Feedback Loop Performance: Dominant Poles

Explanation

- Many practical cases: Dominant conjugated complex pole pair in closed-loop sensitivities
  $\Rightarrow 1 + C(s)\,G(s) = (1 + 2\,D\,T\,s + T^2\,s^2)\,F(s)$ $(0 < D < 1)$
- Closed-loop performance is determined by zeros of $(1 + 2\,D\,T\,s + T^2\,s^2)$

Reminder

- Rise time: $t_r = \frac{T}{\sqrt{1-D^2}} \arctan(\frac{\sqrt{1-D^2}}{D})$
- Peak time: $t_p = \frac{\pi\,T}{\sqrt{1-D^2}}$
- Overshoot: $M_p = e^{-\frac{D}{1-D^2}\pi}$
- Settling time: $t_s = \frac{3\,T}{D}$ (5%); $t_s = \frac{4\,T}{D}$ (2%)

Feedback Loop Performance: Application

Computation

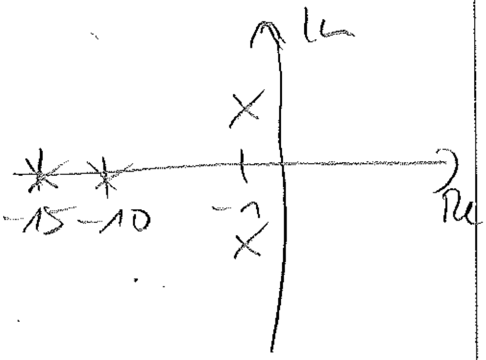
Gap 10

Example: $T(s) = \frac{5}{(s^2+2s+2)(s+10)(s+15)}$

$\Rightarrow$ dominant poles at $p_{1/2} = \frac{1}{2}(-2 \pm j \cdot 2)$

$\Rightarrow T = 1/\sqrt{2}$, $D = 1/\sqrt{2}$

$\Rightarrow$ output response is dominated by $p_{1/2}$



$t_r \approx \frac{1}{\sqrt{2}} \cdot \sqrt{2} \cdot \frac{\pi}{4}$

$t_p \approx \pi$ and $M_p \approx e^{-\pi}$

$t_s \approx 4$ (2%)