ECE 488 – Automatic Control

Observability and Separation

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Weeks

- Plant modeling
- Properties of transfer functions
- Stability and performance
- Feedback control
- Root locus method
- Nyquist criterion and bode plot
- Lead/lag compensator and PID-controller
- Controllability and State feedback control

This week

- Observability
- Separation

Observability: Definition

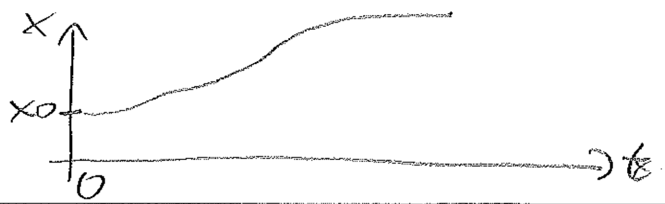
State-space Model

$$\dot{x}(t) = A\,x(t) + b\,u(t)$$
$$y(t) = c^T x(t) + d\,u(t)$$

Observability Definition

A linear system is called <u>*completely observable*</u> *if any initial state $x(0)$ at time $t = 0$ can be uniquely determined from the output signal $y(t)$ and the input signal $u(t)$ in a pre-specified time interval $0 \leq t \leq t_T$.*

Gap 1



observable $y(t)$, control $u(t)$ } find x_0

Observability: Verification

Observability Test by Kalman

A linear system of order n is completely observable if and only if the <u>*observability matrix*</u> $\mathcal{O} = \begin{bmatrix} c^T \\ c^T A \\ \vdots \\ c^T A^{n-1} \end{bmatrix}$ *has full rank n*

Example

Gap 2

$A = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}$ $c^T = (1 \quad 0) \Rightarrow \mathcal{O} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$\mathcal{O}$ has full rank $\Rightarrow$ observable

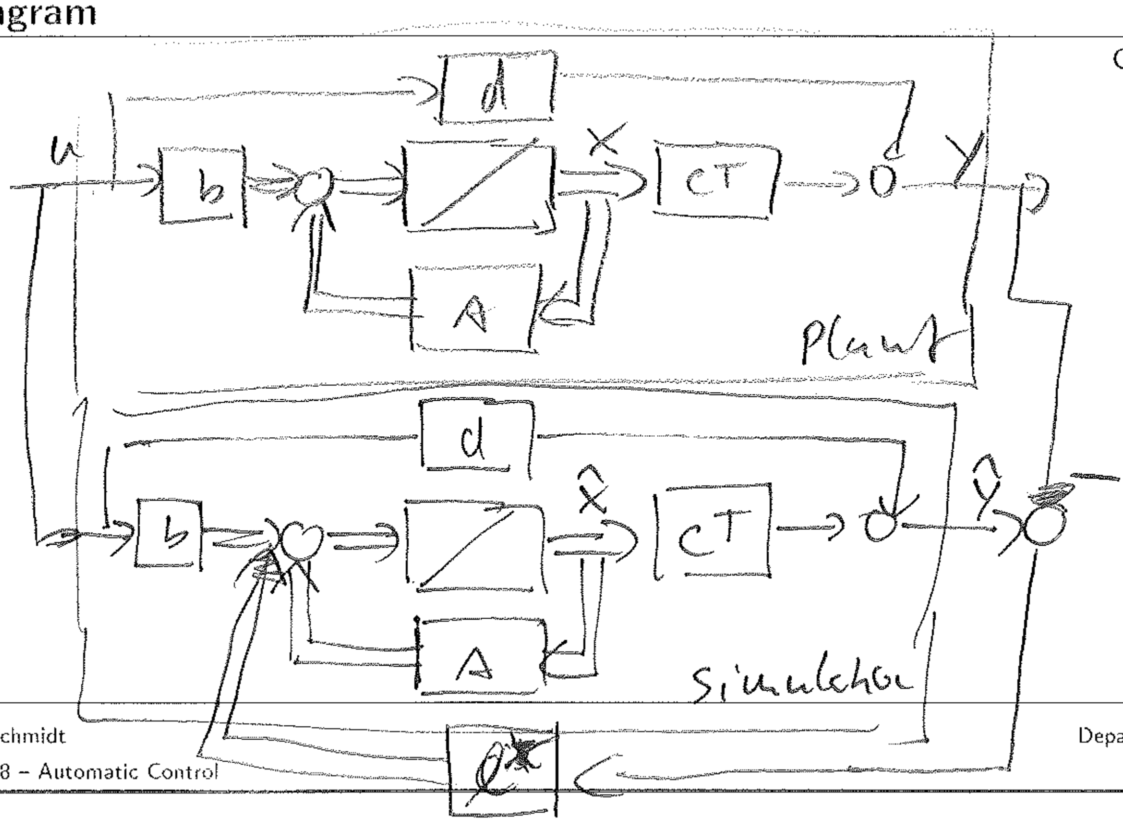
$c^T = (1 \quad 1) \Rightarrow \mathcal{O} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\mathcal{O}$ does not have full rank $\Rightarrow$ not observable

Luenberg Observer: Block Diagram

Diagram

Gap 3



Luenberg Observer: Idea

State Reconstruction

- Parallel model of the linear system (simulated)
- Comparison of measured and simulated output
- Feedback to compensate error

Parallel Model

$$\dot{\hat{x}}(t) = A\,\hat{x}(t) + b\,u(t) + l(\hat{y}(t) - y(t))$$
$$\hat{y}(t) = c^T\,\hat{x}(t) + d\,u(t)$$

Variables and Parameters

- Simulated state $\hat{x}$
- Simulated output $\hat{y}$
- Feedback vector l

Luenberg Observer: Computation

State Error Computation

Gap 4

$\dot{e}(t) = \dot{\hat{x}} - \dot{x} = A\hat{x} + bu + l(\hat{y} - y) - Ax - bu =$

$= A\hat{x} - Ax + l(c^T\hat{x} + du - c^T x - du) =$

$= (A + lc^T)(\hat{x} - x) = (A + lc^T) \cdot e$

Result

$$\dot{e}(t) = \dot{\hat{x}}(t) - \dot{x}(t) = (A + l\,c^T)e(t)$$

- Stabilize observer by moving the eigenvalues of $A + l\,c^T$ in the OLHP
- Desired characteristic polynomial of $A + l\,c^T$: $p(s)$
  $\rightarrow$ We want $\det(sI - A - l\,c^T) = p(s)$

Luenberg Observer: Example

Computation

Gap 5

$A = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}$ $c^T = [1 \quad 0]$; $p_1 = p_2 = -10$

$\Rightarrow p(s) = s^2 + 20s + 100$

Use $l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$

$\det(sI - A - lc^T) = \det\left(\begin{bmatrix} s & -3 \\ -1 & s+2 \end{bmatrix} - \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix}\right) =$

$= \det\begin{bmatrix} s - l_1 & -3 \\ -1-l_2 & s+2 \end{bmatrix} = (s - l_1)(s+2) + 3(-1-l_2) =$

$= s^2 + s(-l_1 + 2) - 2l_1 - 3 - 3l_2 = s^2 + 20s + 100$

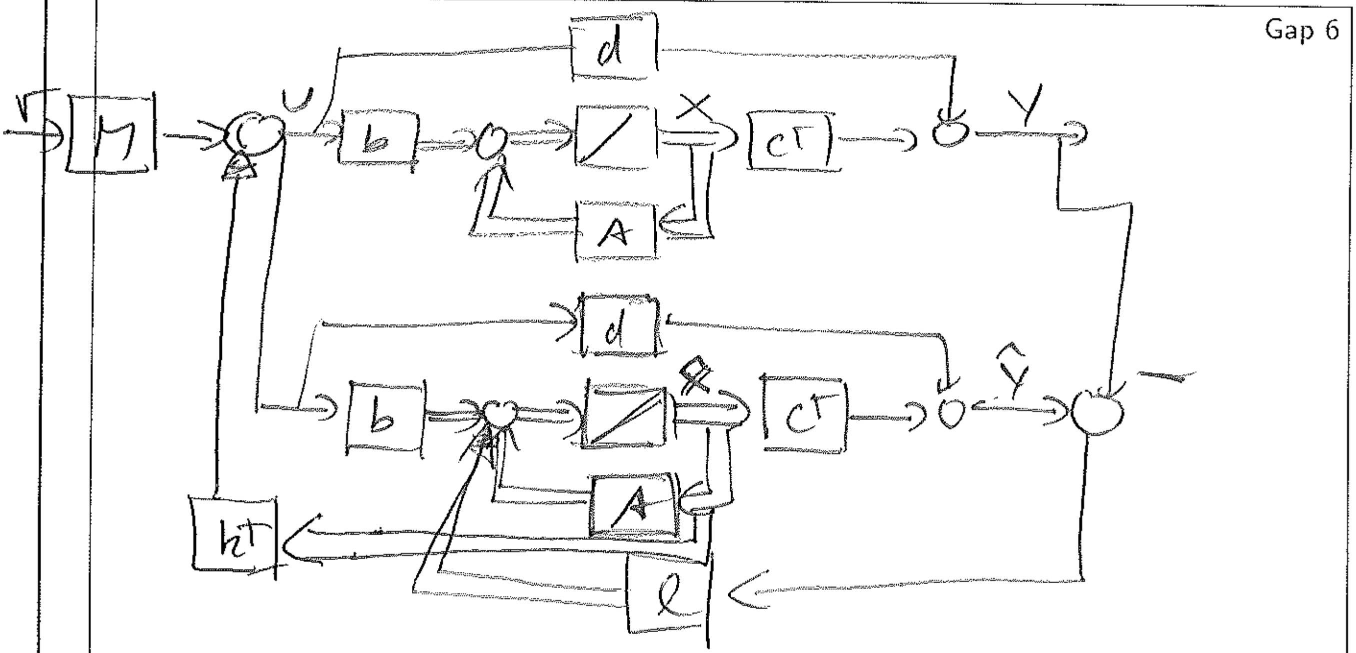
$\Rightarrow l_1 = -18,\ l_2 = \frac{1}{3}(-2l_1 - 3 - 100) = -\frac{67}{3}$

$\Rightarrow l = \begin{bmatrix} -18 \\ -67/3 \end{bmatrix}$

Separation: Combination of State Feedback and Observer

Block Diagram

Gap 6



Separation: Explanation

State Estimation

- Use the estimated state $\hat{x}$ for state feedback
  $\Rightarrow$ Controller consists of observer (for state estimation), state feedback (for pole placement) and pre-filter (for steady-state value)
- Closed loop eigenvalues are eigenvalues of observer plus eigenvalues of state feedback $\rightarrow$ principle of separation

Gap 7

$$u = k^T \hat{x} + M \cdot r$$

$$\dot{x} = Ax + bu = Ax + bk^T\hat{x} + bMr$$

$$\dot{\hat{x}} = A\hat{x} + bu + l(\hat{y} - y) =$$

$$= A\hat{x} + bk^T\hat{x} + bMr + l(c^T\hat{x} + \cancel{du} - c^Tx - \cancel{du})$$

$$= (A + lc^T + bk^T)\hat{x} - lc^Tx + bMr$$

Separation: Example

Computation

Gap 8

$$\begin{bmatrix}\dot{x}\\ \dot{\hat{x}}\end{bmatrix} = \begin{bmatrix}A & bk^T\\ -lc^T & A+bh^T+lc^T\end{bmatrix}\begin{bmatrix}x\\ \hat{x}\end{bmatrix} + \begin{bmatrix}bM\\ bM\end{bmatrix} r$$

State transformation: $\begin{bmatrix}x\\ \hat{x}\end{bmatrix} = Tz$, $T = \begin{bmatrix}I & 0\\ I & I\end{bmatrix}$

$$\Rightarrow \dot{z} = \begin{bmatrix}I & 0\\ -I & I\end{bmatrix}\begin{bmatrix}A & bh^T\\ -lc^T & A+bh^T+lc^T\end{bmatrix} z + T^{-1}\begin{bmatrix}bM\\ bM\end{bmatrix} r = \qquad T^{-1} = \begin{bmatrix}I & 0\\ -I & I\end{bmatrix}$$

$$= \begin{bmatrix}A & bh^T\\ -A-lc^T & A+lc^T\end{bmatrix}\begin{bmatrix}I & 0\\ I & I\end{bmatrix} z + \begin{bmatrix}bM\\ 0\end{bmatrix} r =$$

$$= \begin{bmatrix}A+bh^T & bh^T\\ 0 & A+lc^T\end{bmatrix} z + \begin{bmatrix}bM\\ 0\end{bmatrix} r$$

$\Rightarrow$ eigenvalues of $A+bh^T$ and $A+lc^T$