ECE 488 – Automatic Control

PID Control

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Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Weeks

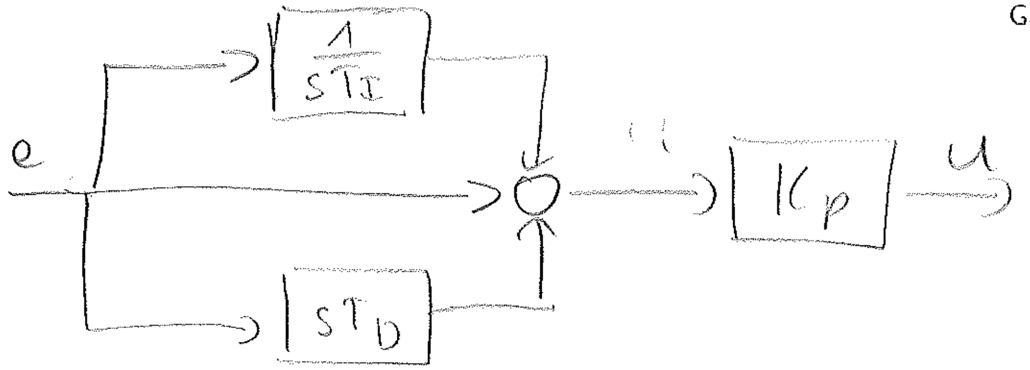
- LTI system modeling
- Nonlinear modeling and linearization
- Stability
- Steady-state and transient response
- Feedback Control
  - Root locus
  - Nyquist plot
- Bode plot and Lead/lag compensation

This week

- PID controller
- PID design

PID Controller: Characteristics

Gap 1



Ordinary Differential Equation (ODE)

$$u = K_p \cdot (e + \frac{1}{T_I}\int e + T_D \dot{e})$$

Transfer Function (TF)

$$U(s) = K_p \cdot (E(s) + \frac{1}{T_I\, s} E(s) + T_D\, s\, E(s) = K_p\,(1 + \frac{1}{T_I\, s} + T_D\, s)\, E(s)$$

PID Controller: Parameters

Proportional Action: $K_p \cdot e$

- Depends on instantaneous value of error
- Can control any stable plant but usually with low performance

Integral Action: $\frac{K_p}{T_I} \cdot \int e$

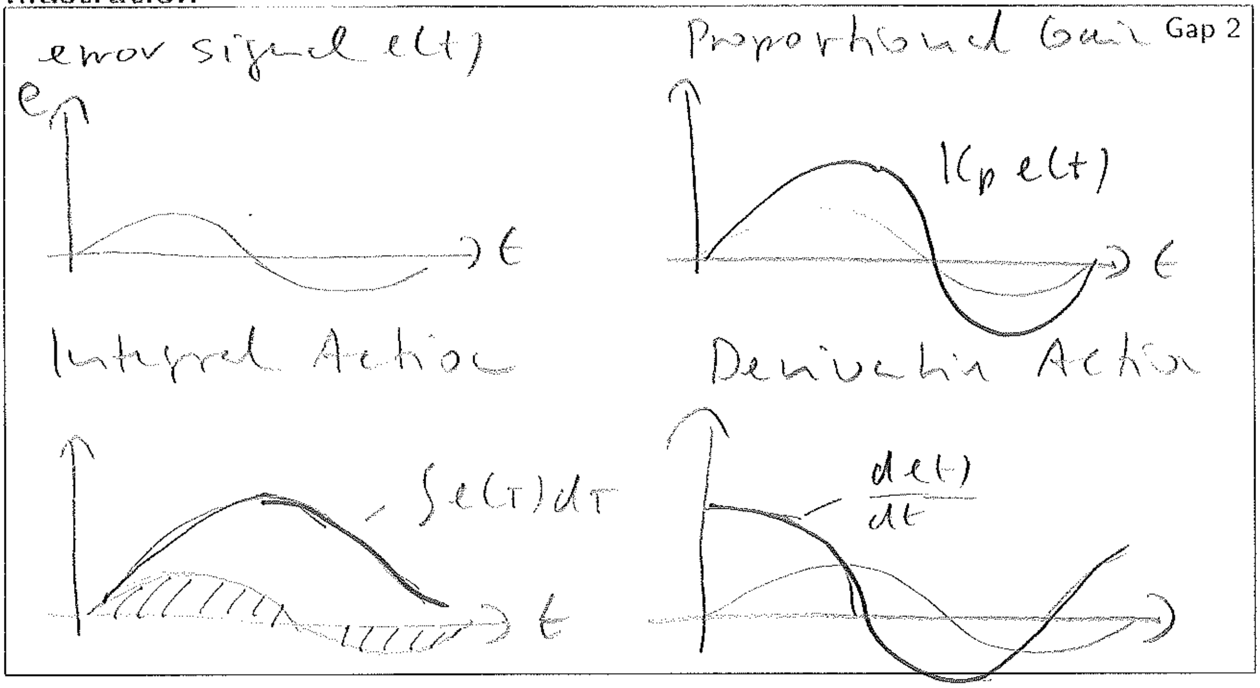
- Realizes memory due to dependency on accumulated error
- Enforces steady state error of $\lim_{t\to\infty} e(t) = 0$

Derivative Action: $K_p T_D \cdot \dot{e}$

- Captures trend of the error due to dependency on rate of change of e
- Susceptible to amplification of high-frequency disturbances/noise

PID Controller: Parameters

Illustration



PID Controller: Special Cases

PI-Controller

$$C(s) = K_p (1 + \frac{1}{T_I s})$$

PD-Controller

$$C(s) = K_p (1 + T_D s)$$

Design Task

- Determine the most suitable controller type and the controller parameters K_p, T_I and T_D in order to fulfill given performance specifications

Ziegler-Nichols: Oscillation Method

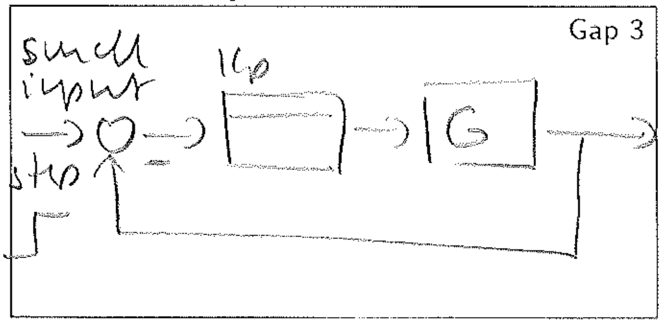
Assumption

- Stable, non-oscillatory plant: $G(s) = K\dfrac{e^{-s\tau}}{(1+sT_1)\cdots(1+sT_n)}$
  excluding first-order/second-order lag
- Note: plant is not modeled!

Practical Experiment

- Start with $K_p = 0$ and increase K_p gradually until y oscillates
- Note critical gain $K_{\mathrm{crit}} = K_p$
- Note oscillation period T_{crit}

Control Loop with P-control



Ziegler-Nichols: Oscillation Method

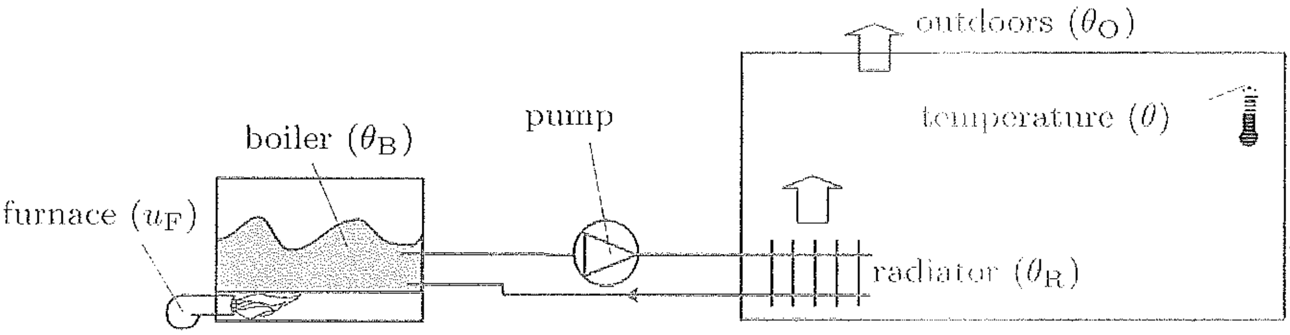
PID-controller Parameters

Controller	K_p	T_I	T_D
P-	$0.5K_{\mathrm{crit}}$	∞	0
PI-	$0.45K_{\mathrm{crit}}$	$0.85\,T_{\mathrm{crit}}$	0
PID-	$0.6K_{\mathrm{crit}}$	$0.5\,T_{\mathrm{crit}}$	$0.12\,T_{\mathrm{crit}}$

Results

- Stable closed loop
- Addresses both reference tracking and disturbance rejection

Example (temperature control)



Ziegler-Nichols: Example

Room temperature example

Gap 4

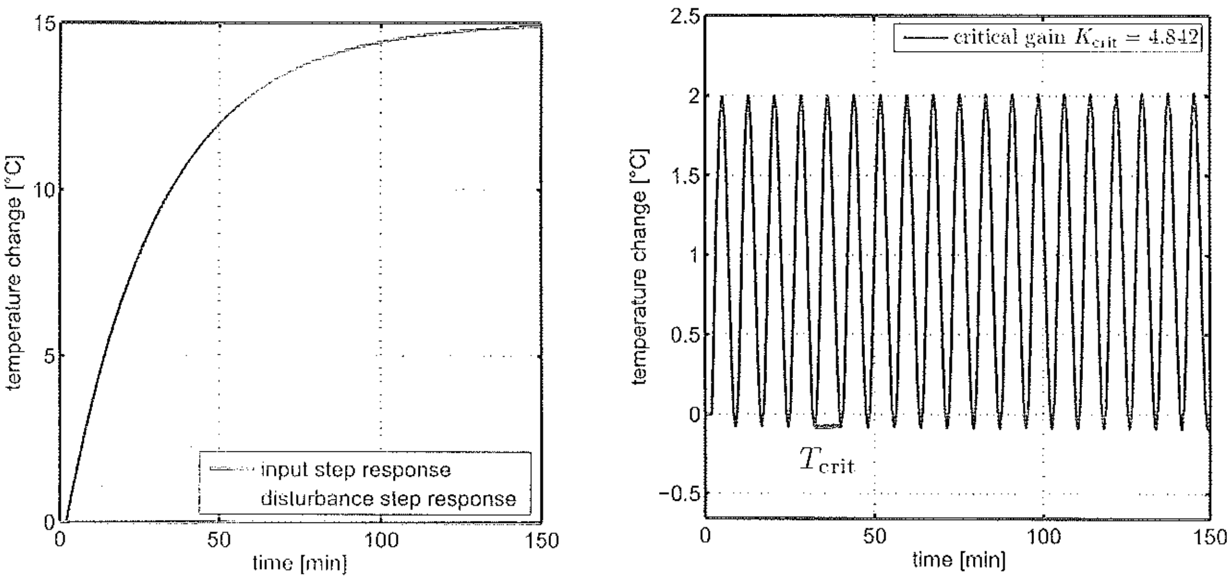
Read $K_{crit} = 4.8$ | $T_{crit} = 3.8$

P-control	PI-control	PID-control
$K_p = 2.4$	$K_p = 2.2$	$K_p = 2.9$
$T_I = \infty$	$T_I = 6.6$	$T_I = 3.9$
$T_D = 0$	$T_D = 0$	$T_D = 0.9$

Ziegler-Nichols: Oscillation Method

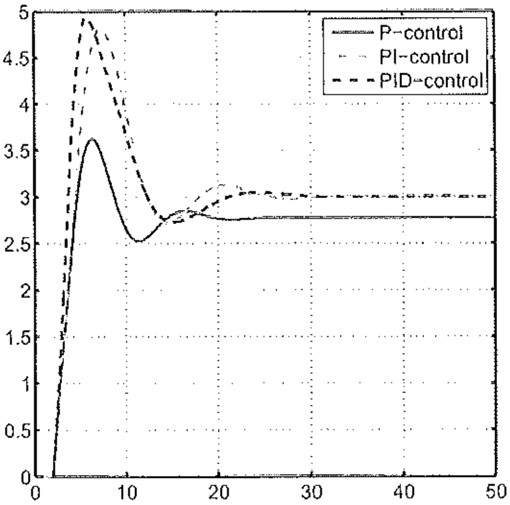
Uncontrolled Plant Step Response

Oscillation Experiment

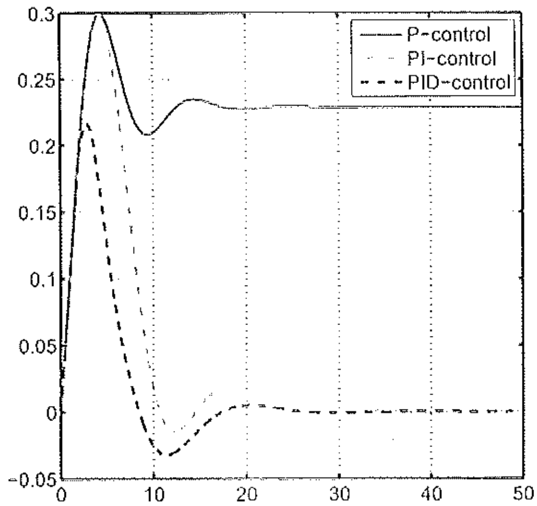


Ziegler-Nichols: Oscillation Method

Reference Step Response



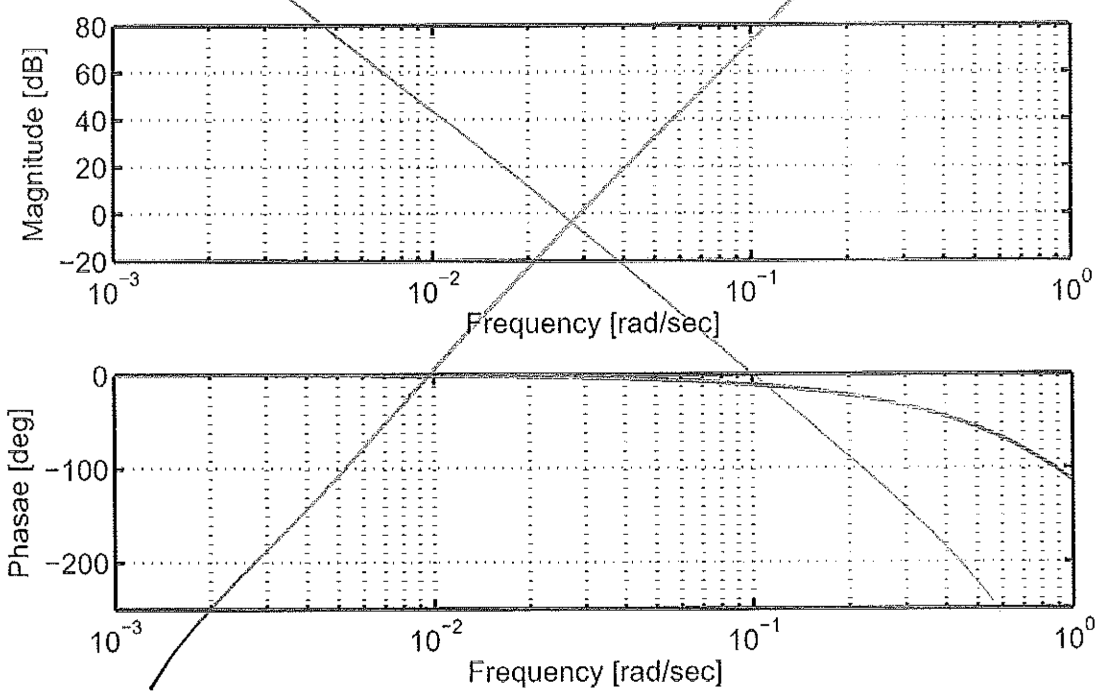
Disturbance Step Response



⇒ No exact following with P-control
⇒ Larger overshoot for PI and PID control due to plant delay
⇒ Similar dynamics for reference tracking and disturbance rejection

Ziegler-Nichols: Bode Plot

PI, show plot



Ziegler-Nichols: Example

Computation

Gap 5

$\Phi_m = 35°$

$\omega_g = 0.45$ rad/sec

⇒ stable but oscillatory, lightly damped

Remarks

- Parameter computation for stability and sufficient phase margin
- Bandwidth of the closed loop according to gain crossover frequency
- Slope around the gain crossover frequency is −20 dB
- Further tuning of parameters is possible for example with Bode plot

Ziegler-Nichols: Example

Computation

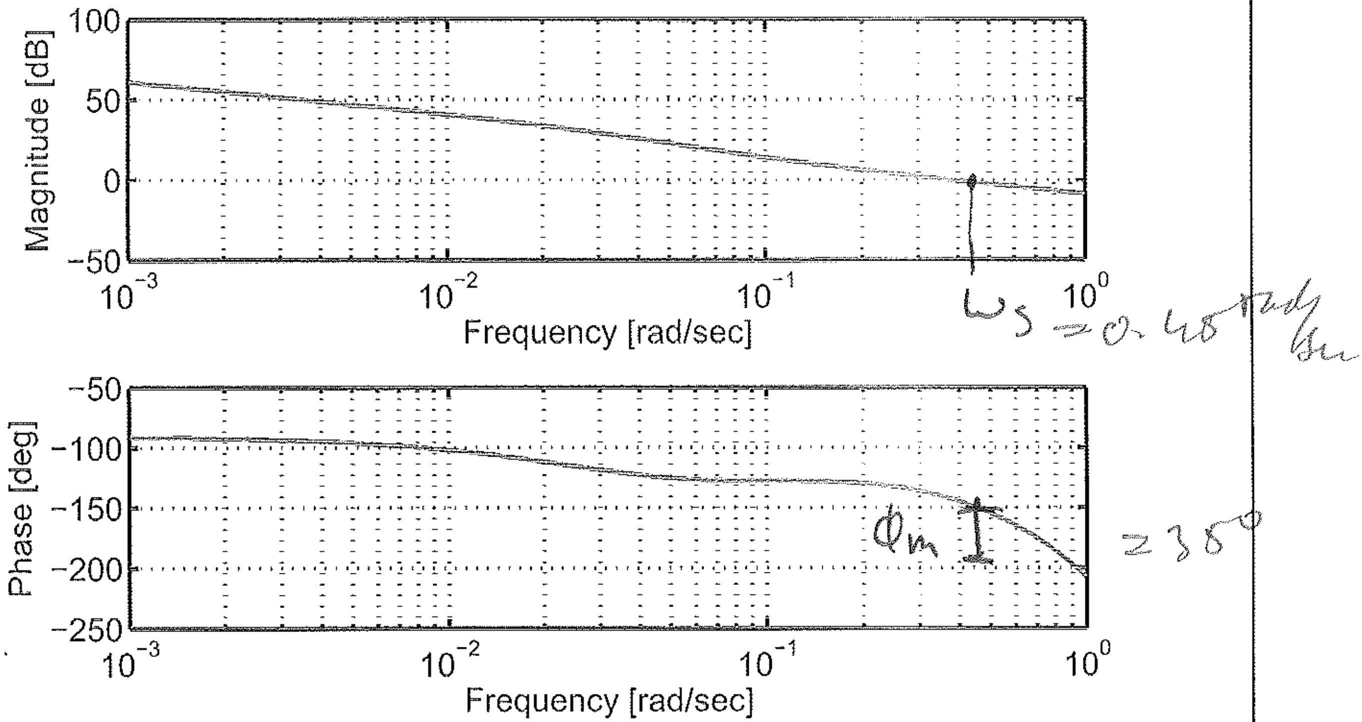
Gap 6

Idea: decrease ω_g to increase Φ_m

⇒ for example decrease gain and increase T_I

$C(s) = \frac{1+10s}{10s}$

Ziegler-Nichols: Bode Plot



Ziegler-Nichols: Reaction Curve Method

Assumption

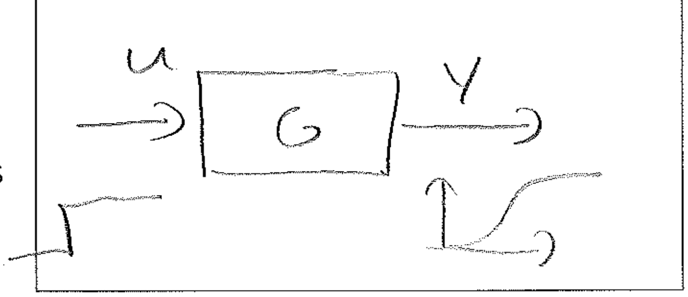
- Stable, non-oscillatory plant: $G(s) = K\frac{e^{-sT}}{(1+sT_1)\cdots(1+sT_n)}$
  excluding $G(s) = \frac{K}{1+sT_1}$
- Note: plant is not modeled!

Practical Experiment

- Approach desired operating point
- Apply "small" step input
- Record plant output: process reaction curve

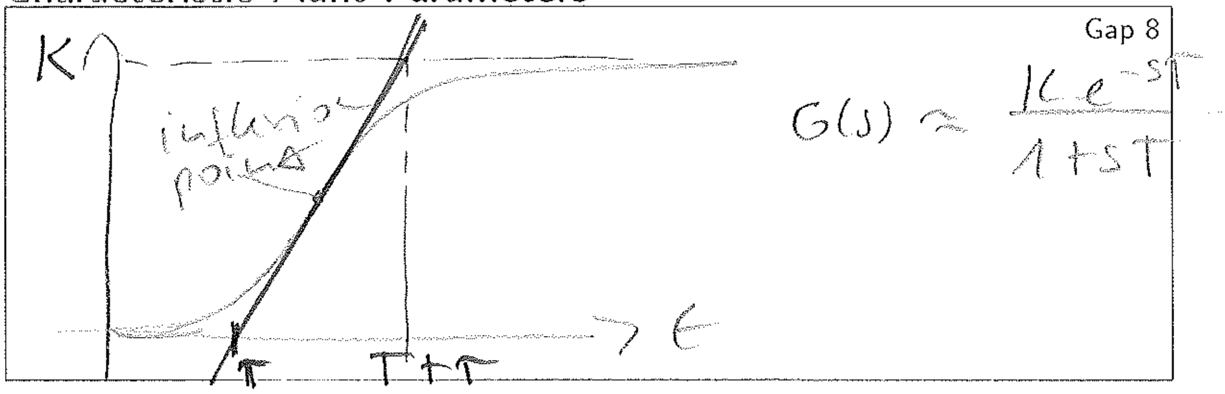
Step Response in Open Loop

Gap 7



Ziegler-Nichols: Reaction Curve Method

Characteristic Plant Parameters

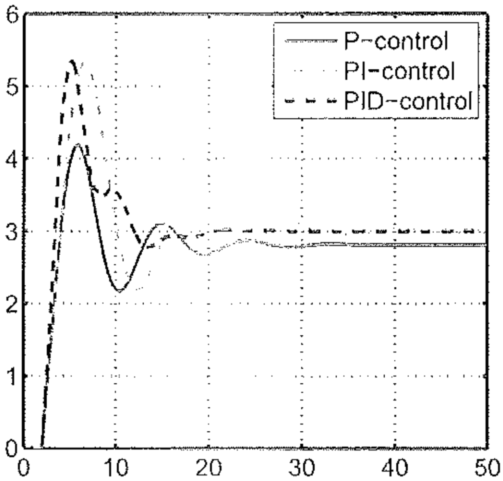


PID-controller Parameters

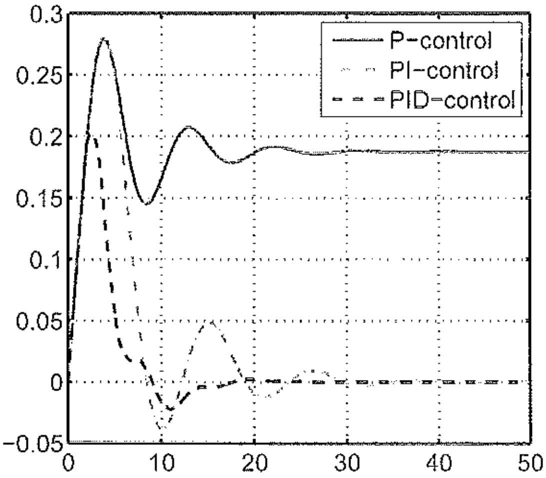
Controller	K_p	T_I	T_D
P-	$1/K \cdot T/\tau$	∞	0
PI-	$0.9/K \cdot T/\tau$	3.33τ	0
PID-	$1.2/K \cdot T/\tau$	2τ	0.5τ

Ziegler-Nichols: Oscillation Method

Reference Step Response



Disturbance Step Response



$\Rightarrow$ Similar behavior to Ziegler-Nichols Oscillation Method

Ziegler-Nichols: Concluding Remarks

Usage

- Stable non-oscillatory plants
- No mathematical model required
- Oscillation or step response experiments
- Design of P, PI and PID-controllers
- Stable closed loop

Limitations

- Requires certain plant characteristics
- Empirical design should be followed by fine tuning on physical system
- Usage of integral control can lead to large overshoot for delay plants

More PID Design Methods

- For example, ECE 441, ECE 438