ECE 488 – Automatic Control

Bode Plot Design

Assistant Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineeering – Çankaya University

Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Reminder

Previous Weeks

- LTI system modeling
- Nonlinear modeling and linearization
- Stability
- Steady-state and transient response
- Feedback Control
  - Root locus
  - Nyquist plot
- Bode plot

This week

- Bode plot design
- Lead-lag compensation

Closed-loop Requirements: Motivation

Open-loop Transfer Function

$$G_o(s) = C(s)\, G(s)$$

Complementary Sensitivity

$$T(s) = \frac{C(s)\, G(s)}{1 + C(s)\, G(s)} = \frac{G_o(s)}{1 + G_o(s)}$$

Goal

- Bode plot of $G_o(s) = C(s)\, G(s)$ can be easily obtained
- Modification of $C(s)$ appears as addition of numerator/denominator factors in the bode plot of $G_o(s)$
- Effect of modification of $C(s)$ on $T(s)$ is not as clear
  $\Rightarrow$ Relate properties of Bode plot of $G_o(s)$ to properties of $T(s)$

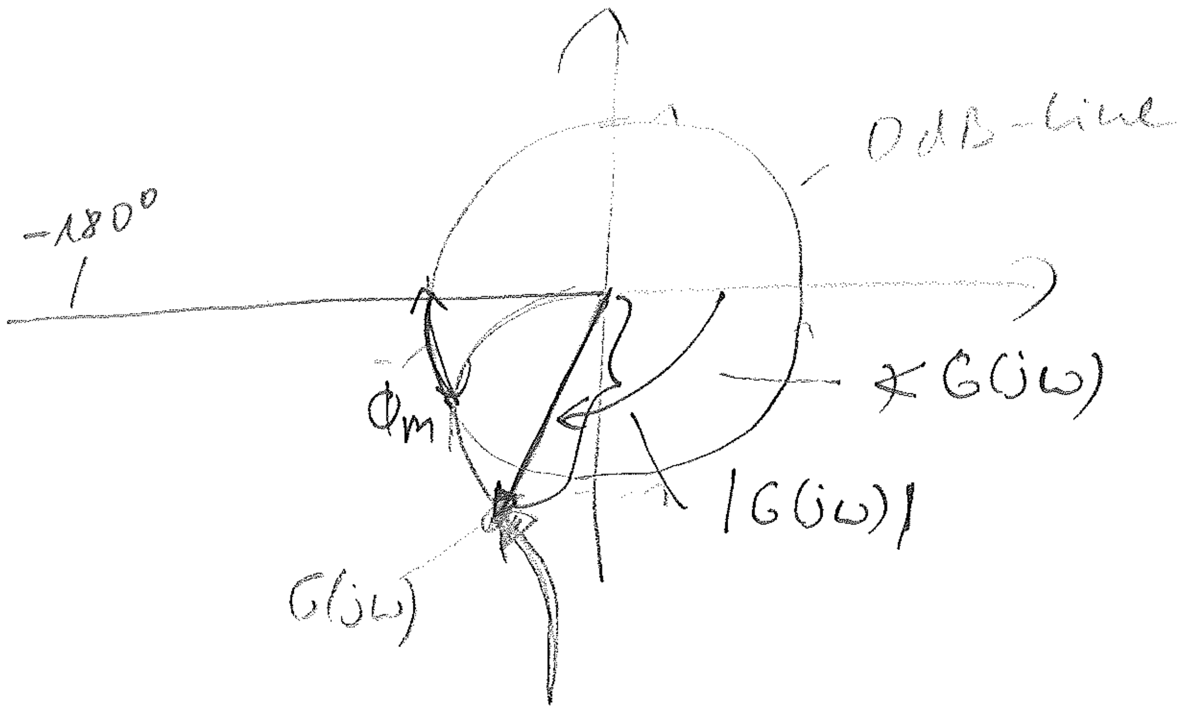
Closed-loop Requirements: Dominant Complex Pole Pair

Approximated Closed-loop Transfer Function

$$T(s) = \frac{K}{1 + 2\, D\, T\, s + T^2\, s^2}$$

Performance Parameters

- DC Gain: $K_{DC} = K$
  $\Rightarrow$ Determines final value of the unit step response
- Overshoot: $M_p = e^{-\frac{D\,\pi}{\sqrt{1-D^2}}}$
- Phase margin: $\Phi_m = \arctan\left(\frac{2\,D}{\sqrt{-2\,D^2 + \sqrt{1 + 4\,D^4}}}\right)$
  $\Rightarrow$ Determines amplitude of oscillations and relative stability
- Cut-off frequency: $\omega_c = \frac{1}{T}\sqrt{1 - 2\,D^2 + \sqrt{(1 - 2\,D^2)^2 + 1}}$
  $\Rightarrow$ Determines speed of response



0dB-Line
-180°
Φ_m
$\measuredangle G(j\omega)$
$|G(j\omega)|$
$G(j\omega)$

HP LaserJet P3005 printers

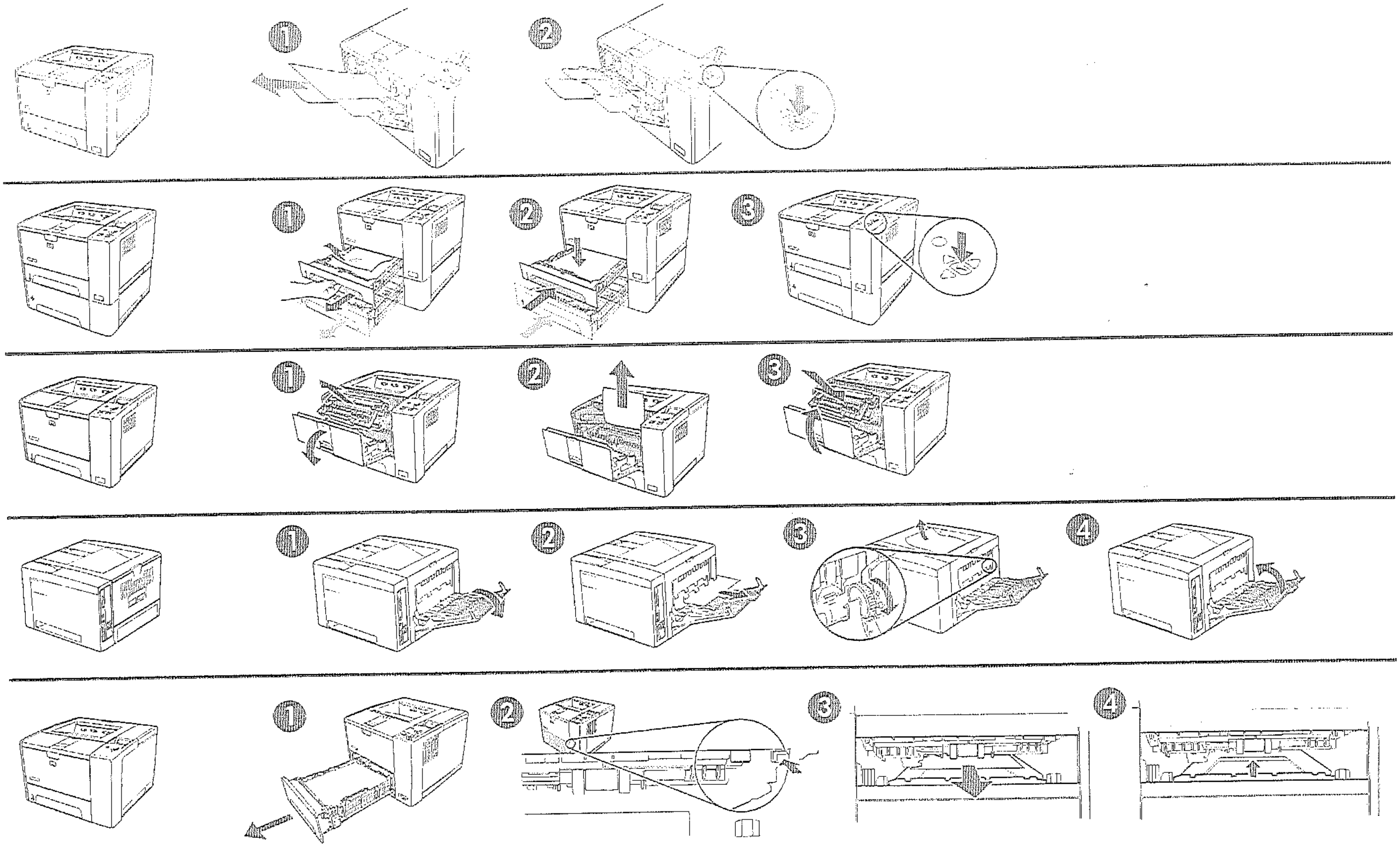
Clear Jams
清除卡纸
Elakadások elhárítása
Устранение замятий

Eliminer les bourrages
Odstranění uvíznutých médií
용지 걸림 해결
Rensa trassel

Staus beseitigen
Afhjælp papirstop
Fjerne fastkjørt papir
清除卡紙

Eliminazione degli inceppamenti
Storingen verhelpen
Usuwanie zacięć
Kağıt Sıkışmalarını Giderme

Eliminación de atascos
Tukosten poistaminen
Limpar atolamentos

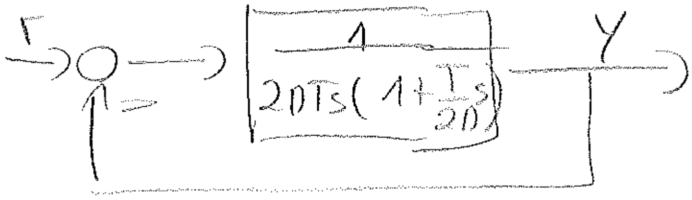


Closed-loop Requirements: Performance Specification

Desired Performance

- DC Gain: $K \approx 1$

Gap 1

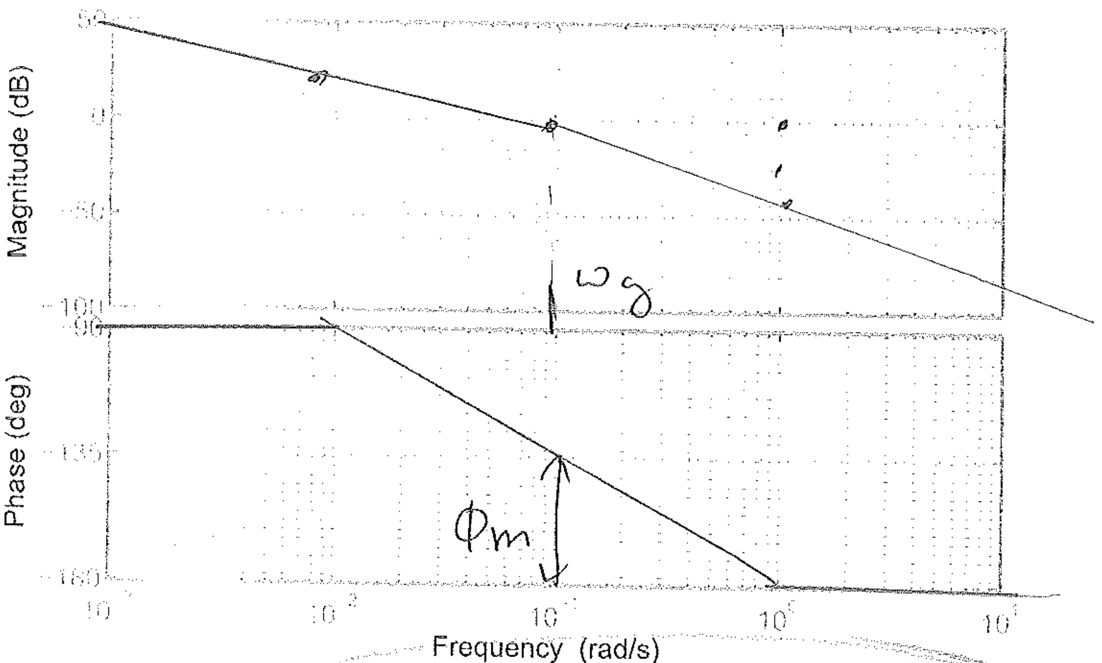


- Open-loop transfer function: $G_o(s) = \frac{1}{2\,D\,T}\frac{1}{s\,(1+\frac{T}{2\,D}\,s)}$
- Sufficient relative stability and small overshoot:
  $\Rightarrow$ Appropriate choice of D. Generally, $D > 0.4$
- Sufficient speed of response:
  $\Rightarrow$ Appropriate choice of ω_c

Frequency Response: Open-loop Bode Plot

D = 0.5, T = 10

$G_o(s) = \frac{0.1}{s(1+10s)}$

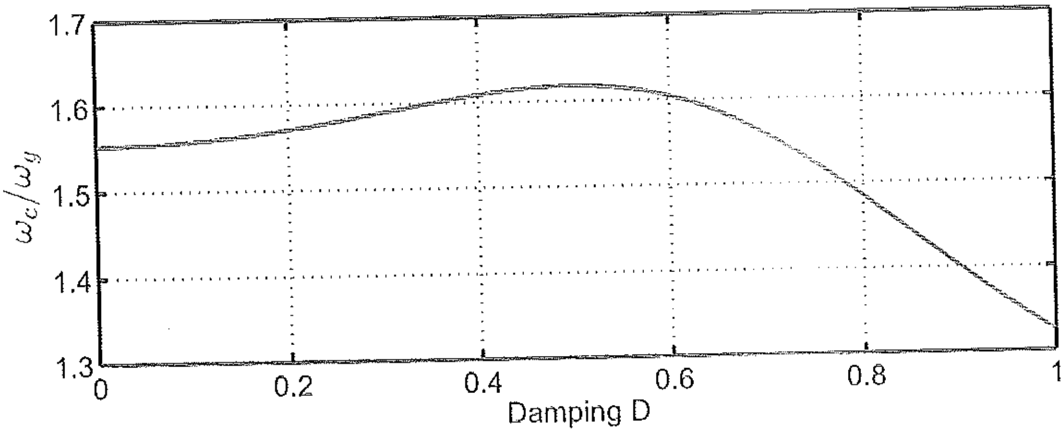


add relation to Nyquist

Closed-loop Requirements: Open-loop Transfer Function

Parameters in Bode Diagram

- Gain crossover frequency: $\omega_g = \frac{1}{T}\sqrt{-2\,D^2 + \sqrt{1 + 4\,D^4}}$
  $\Rightarrow$ It holds the $\omega_g \approx \omega_c$



- Phase margin: Φ_m
  $\Rightarrow$ Compute from stability requirement. Usually, $\Phi_m > 40°$

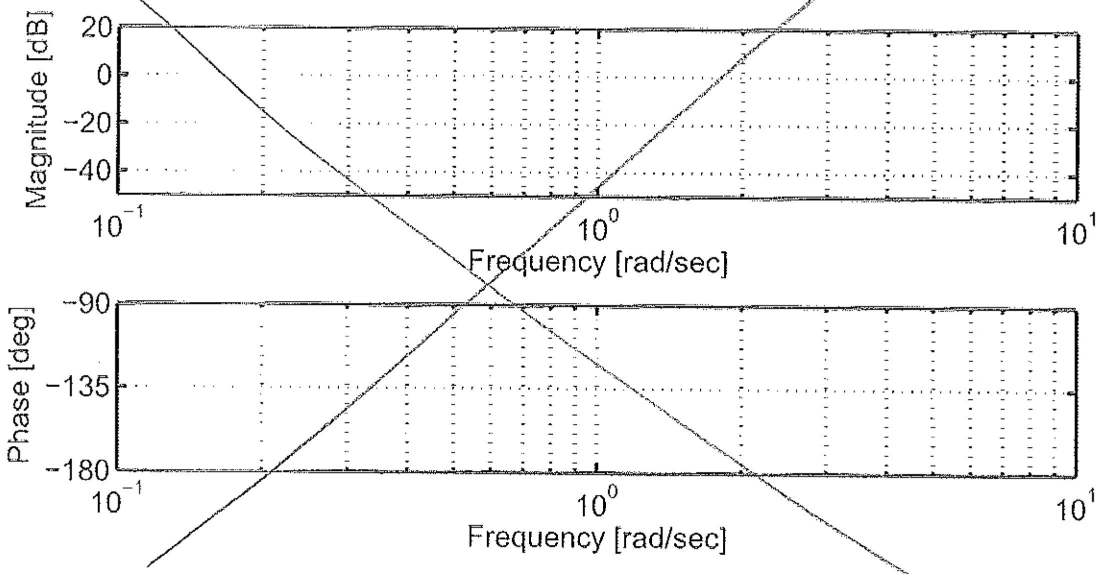
Closed-loop Requirements: Basic Control Design

Procedure

- Determine the closed-loop requirements
  $\Rightarrow$ ω_c and Φ_m
- Determine the desired open-loop transfer function $G_o(s)$
  $\Rightarrow$ $\omega_g \approx \omega_c$ and Φ_m
- Determine the Bode plot of the plant transfer function $G(s)$
  $\Rightarrow$ Fixed part of the control system
- Design the controller transfer function $C(s)$.
  $\Rightarrow$ Choose the controller poles, zeros and gain such that Bode plot of $C(s)\,G(s)$ approximately matches $G_o(s)$ around the frequency ω_g

Closed-loop Requirements: Example

Computation $G(s) = \frac{0.1}{s\,(s+1)}$ and proportional control $K_p = 10$



⇒ Closed-loop poles at $-0.5 \pm 0.87\,j$

Lead Compensator: Transfer Function

Time-constant Representation

$$C(s) = K_\alpha \, \frac{1 + T\,s}{1 + \alpha\,T\,s}$$

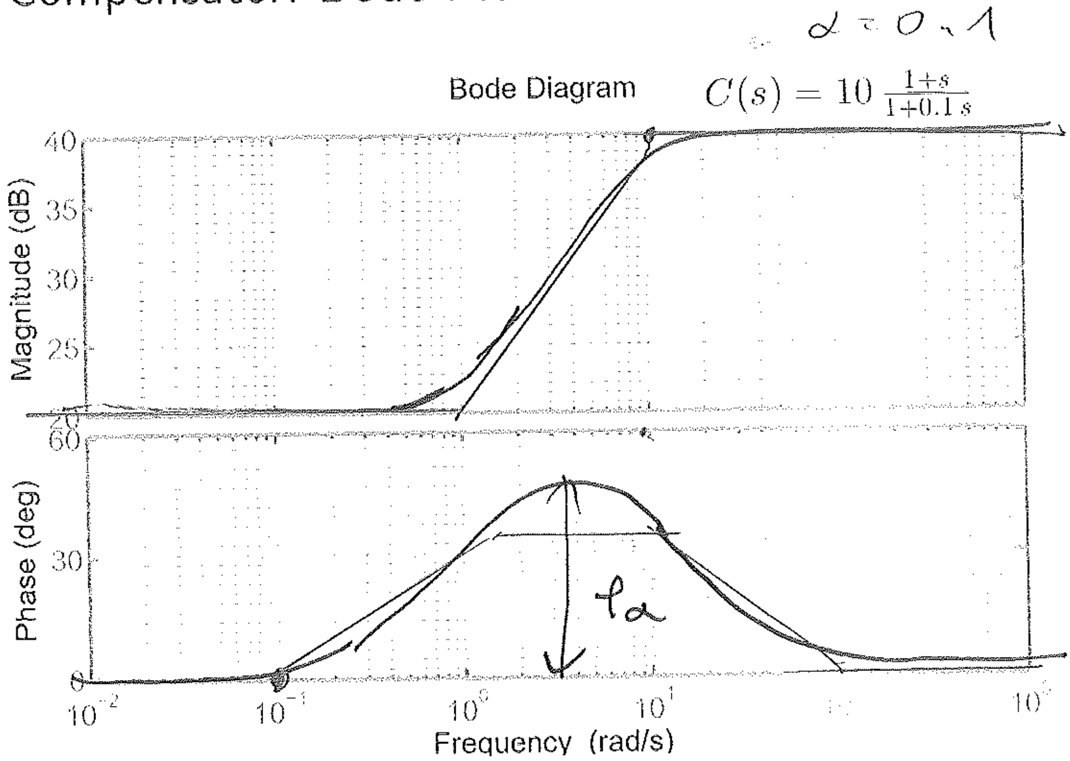
Explanation

- Attenuation factor $0 < \alpha < 1$
- Gain K_α
- Pole at $s = -\frac{1}{\alpha\,T}$
- Zero at $s = -\frac{1}{T}$

Remarks

- Phase increase (lead) up to a maximum value of $\sin(\varphi_\alpha) = \frac{1-\alpha}{1+\alpha}$
- Frequency of maximum phase lead at $\omega_\alpha = \frac{1}{\sqrt{\alpha}\,T}$
- Magnitude at ω_α is $|C(j\omega_\alpha)| = \frac{1}{\sqrt{\alpha}} > 1$

Lead Compensator: Bode Plot



Lead Compensator: Usage

Goal

- Reshape frequency response curve to give additional phase lead in order to increase the phase margin

Starting Point

- Plant transfer function $G(s)$
- Lead compensator transfer function $C(s) = K_\alpha \dfrac{1+T\,s}{1+\alpha\,T\,s}$
- Desired phase margin Φ_m
- Steady-state error e_∞

Task

- Determine the parameters K_α, α and T

Lead Compensator: Procedure

1. Determine the gain K_α to achieve the static error specification
2. Draw a Bode plot of $K_\alpha\, G(j\omega)$ and determine the phase margin Φ'_m
3. Determine the required lead angle $\varphi_\alpha = \Phi_m - \Phi'_m + 10°$
   $\Rightarrow \alpha = \dfrac{1-\sin(\Phi_{max})}{1+\sin(\Phi_{max})}$
4. Choose the gain crossover frequency ω_g such that
   $|K_\alpha\, G(j\omega_g)|_{dB} = -20\log(\frac{1}{\sqrt{\alpha}})$
   $\Rightarrow$ Choose $\omega_\alpha = \omega_g$
5. Evaluate $\omega_\alpha = \frac{1}{\sqrt{\alpha}\, T}$
   $\Rightarrow T = \dfrac{1}{\omega_\alpha\sqrt{\alpha}}$
6. Verify if the design fulfills the specified requirements. Go back to step 3. if the requirements are not fulfilled

Lead Compensator: Example

Computation $G(s) = \frac{4}{s(s+2)}$

Gap 2

We want: static velocity error is $\frac{1}{20}$
Phase margin $\Phi_m = 50°$

1. $\lim_{s\to 0} s\,\frac{1}{1+G(s)\cdot C(s)}\cdot\frac{1}{s^2} = \lim_{s\to\infty}\frac{1}{1+\frac{4}{s\cdot 2}\cdot K_\alpha}\cdot\frac{1}{s} =$
   $= \lim_{s\to\infty}\frac{2s}{2s+4K_\alpha}\cdot\frac{1}{s} = \frac{1}{2K_\alpha} = \frac{1}{20} \Rightarrow K_\alpha = 10$
2. $\Rightarrow$ see bode plot
3. $\Phi'_m = 10°$ (from bode plot)
   $\Rightarrow$ we need $\varphi_\alpha = \Phi_m - \Phi'_m + 10° = 40°$
   $\Rightarrow \alpha = 0.2$

Lead Compensator: Example

Computation

Gap 3

4. $-20 \log\left(\frac{1}{\sqrt{\alpha}}\right) = -7\,dB$

$\Rightarrow \omega_a = 9$ rad/sec

5. $T = \frac{1}{\omega_a \sqrt{\alpha}} = \frac{1}{9\sqrt{0.2}}$ sec $= 0.23$ sec

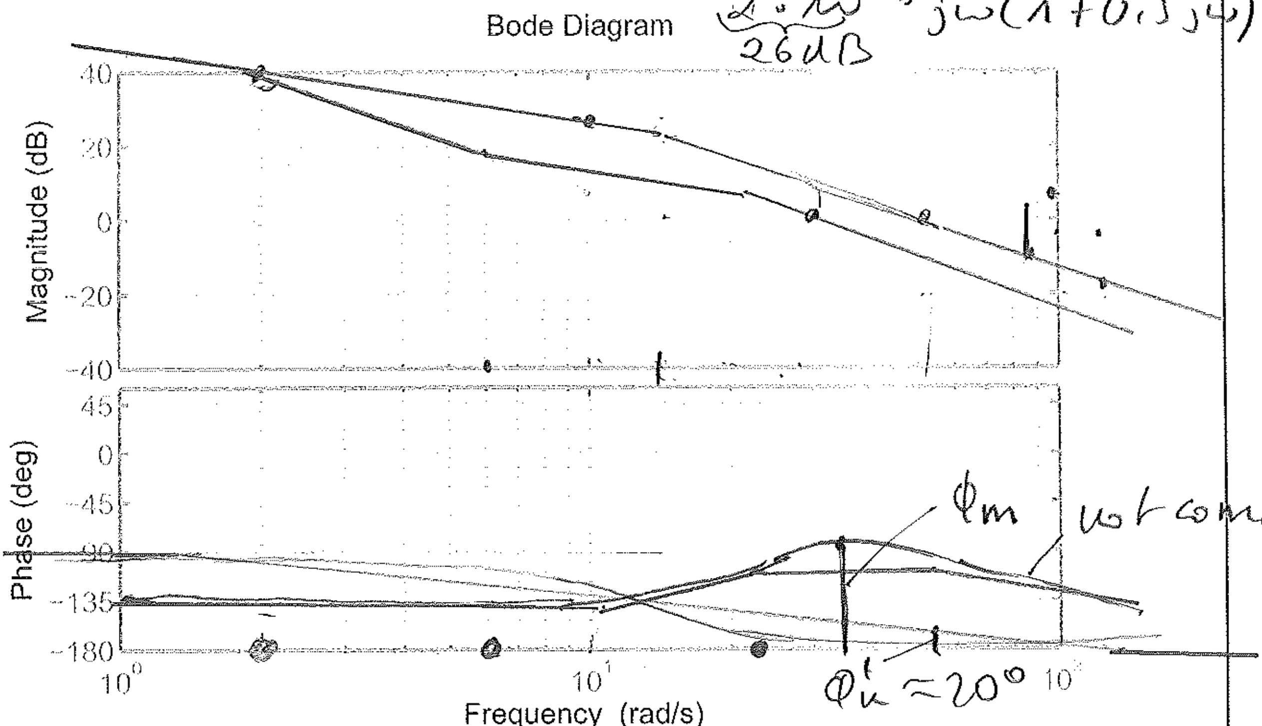
6. Modify Bode plot

$$C(s) = 10 \cdot \frac{1 + 0.23\, s}{1 + 0.045\, s}$$

$$\Rightarrow G_o(j\omega) = 200 \cdot \frac{1 + 0.23 j\omega}{1 + 0.045 j\omega} \cdot \frac{1}{j\omega(1 + 0.5 j\omega)}$$

shift G to left ?!

Lead Compensator: Example



$\Rightarrow$ put plot ?!

Lag Compensator: Transfer Function

Time-constant Representation

$$C(s) = K_\beta \frac{1 + T s}{1 + \beta T s}$$

Explanation

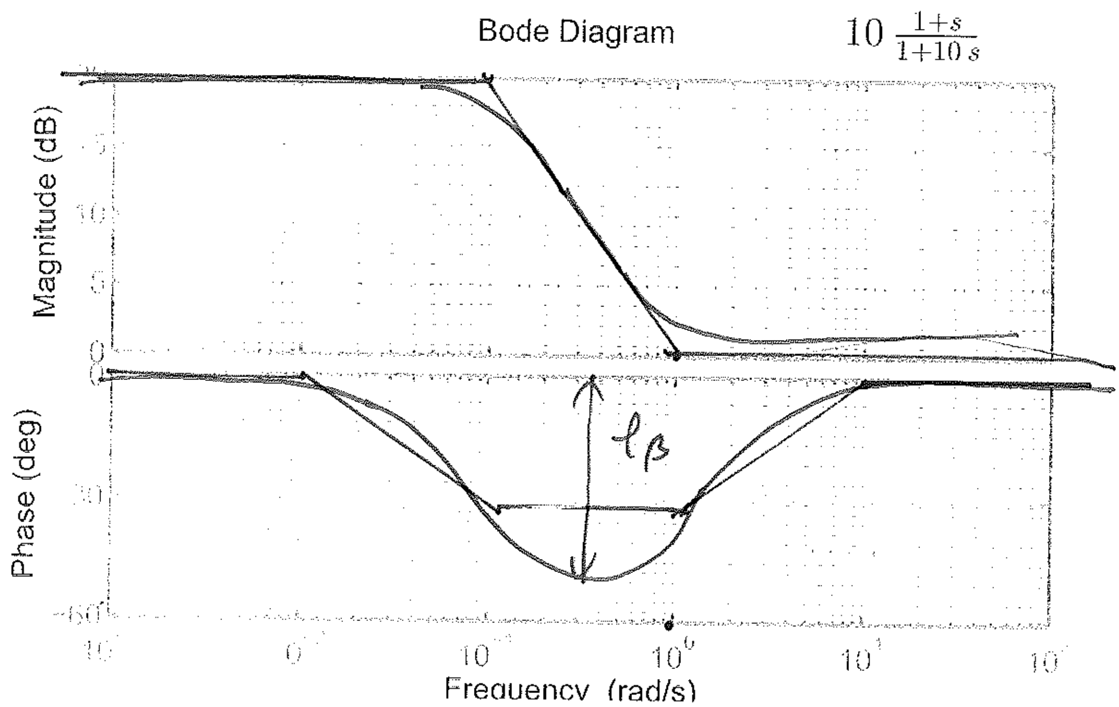
- Attenuation factor $\beta > 1$
- Gain K_β
- Pole at $s = -\frac{1}{\beta T}$
- Zero at $s = -\frac{1}{T}$

Remarks

- Phase decrease (lag) up to a maximum value of $\sin(\varphi_\beta) = \frac{\beta-1}{1+\beta}$
- Frequency of maximum phase lag at $\omega_\beta = \frac{1}{\sqrt{\beta}\, T}$
- Magnitude at ω_β is $|C(j\omega_\beta)| = \frac{1}{\sqrt{\beta}}$

Lag Compensator: Bode Plot

$\beta = 10$



Lag Compensator: Usage

Goal

- Reshape frequency response curve to give attenuation in the high-frequency range in order to provide enough phase margin

Starting Point

- Plant transfer function $G(s)$
- Lag compensator transfer function $C(s) = K_\beta \frac{1+Ts}{1+\beta T s}$
- Desired phase margin Φ_m
- Steady-state error e_∞

Task

- Determine the parameters K_β, β and T

Lag Compensator: Procedure

1. Determine the gain K_β to achieve the static error specification
2. Draw a Bode plot of $K_\beta\, G(j\omega)$ and determine the phase margin Φ'_m
3. Find the gain crossover frequency ω_g, where the phase angle is equal to $-180^\circ + \Phi_m + 10^\circ$
4. Choose $\frac{1}{T}$ one octave (factor 2) or better one decade (factor 10) below ω_g
   $\Rightarrow$ Phase lag does not affect gain crossover frequency
5. Determine β such that $20\log(\beta) = |K_\beta\, G(j\omega)|_{dB}$
   $\Rightarrow$ Magnitude curve crosses the 0-dB line at ω_g
6. Verify if the design fulfills the specified requirements. Go back to step 3. if the requirements are not fulfilled

Lag Compensator: Example

→ move to Homework!

Computation $G(s) = \frac{5/4}{(1+10s)(1+4s)}$

Gap 4

Specification:
- static position error $e_0 = 1/11$
- phase margin: $\Phi_m = 45°$

Steps:

1. $e_0 = \lim_{s \to 0} \frac{1}{1+G_0} = \lim_{s \to 0} \frac{1}{1+K_\beta \cdot \frac{5/4}{(1+10s)(1+4s)}}$
   $= \frac{1}{1+5/4 \cdot K_\beta} \overset{!}{=} \frac{1}{11} \Rightarrow K_\beta = 8$
2. $K_\beta G(s) = \frac{10}{(1+10s)(1+4s)}$
3. phase angle: $-180° + \Phi_m + 10° = -125°$
   $\Rightarrow \omega_g = 0.3$ rad/sec

Lag Compensator: Example

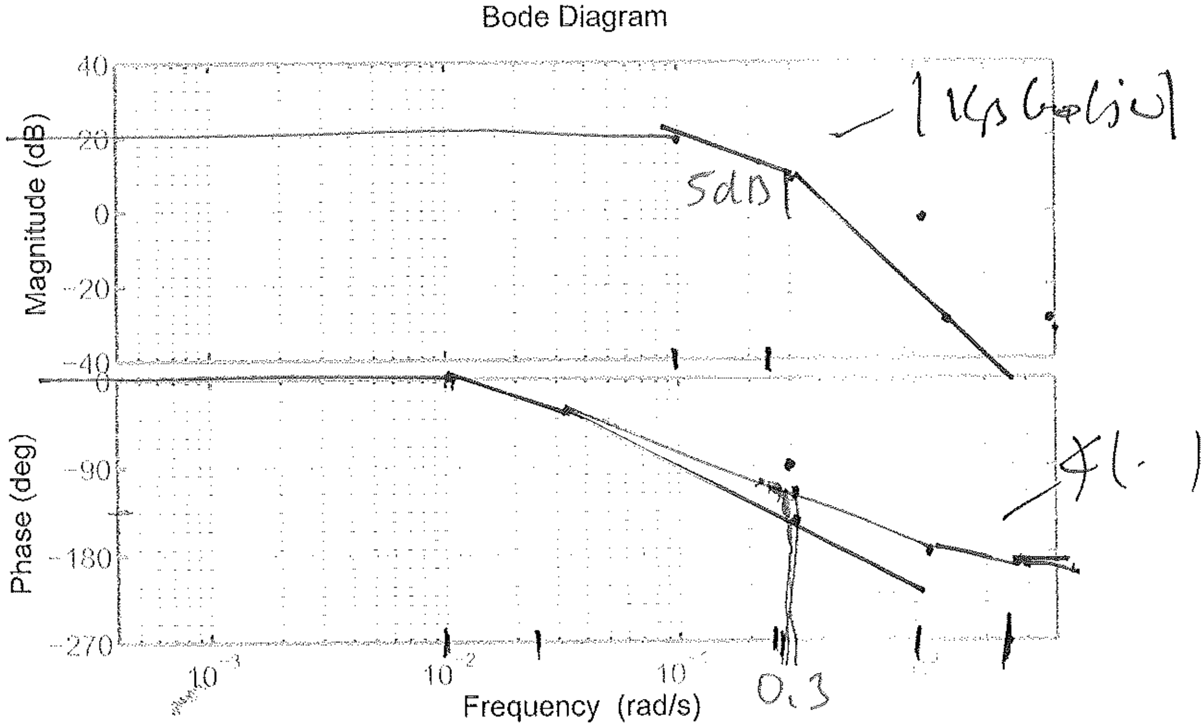
Computation

Gap 5

4. choose $\frac{1}{T} = 0.03 = 0.1 \cdot \omega_g \Rightarrow T = 33$ sec
5. $20 \log(\beta) = |K_\beta G(j\omega_g)| \approx 5$ dB
   $\Rightarrow \beta = 10^{5/20} \approx 1.8$

Together: $C(s) = 8 \cdot \frac{1+33 \cdot s}{1+1.8 \cdot 33 s}$

Lag Compensator: Example



Lag-Lead Compensator: Explanation

Transfer Function

$$C(s) = K \frac{1 + T s}{1 + \alpha T s} \frac{1 + T s}{1 + \beta T s}$$

- $0 < \alpha < 1$ and $\beta > 1$

Description

- Combines advantages of lead compensator and lag compensator
- No detailed discussion in this lecture
- See for example textbook by Ogata, Section 9-4