ECE 488 – Automatic Control

Basics and Plant Modeling

Assistant Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Compulsory Course in Electronic and Communication Engineering
Credits (3/0/3)

Course Webpage: http://ECE488.cankaya.edu.tr

Content and Structure

Content

- Modeling Dynamic Systems
- Linear State Space and Transfer Function Models
- Model Analysis
- Basic Control Concepts
- Control Loop Analysis
- Control Loop Design
- Matlab/Simulink Examples

Structure

- 3 lecture hours: Thursday 15:40 – 17:30; Friday 13:40
- Office hours: Friday 12:40 - 13:30

Grading and Literature

Grading

- 10 Quizzes (10%)
- 12 Homeworks (25%)
- 1 Midterm Exam (25%)
- 1 Final Exam (40 %)

Literature

- Goodwin, Graham, Graebe Stefan, Salgado, Mario: "Control System Design", Prentice-Hall, Inc., 2001 (ISBN: 0-13-958653-9) (Main Textbook)
- Ogata, Katsuhiko: "Modern Control Engineering (5th Edition)", Prentice-Hall, Inc., 2009 (ISBN: 0-13-615673-8)
- Astrom, Karl and Murray, Richard: "Feedback Systems: An Introduction for Scientists and Engineers", Princeton University Press, 2008 (ISBN: 0-691-13576-2)

Motivation: Basics

Control

Control is a discipline that is concerned with the automatic manipulation of a dynamic system's behavior

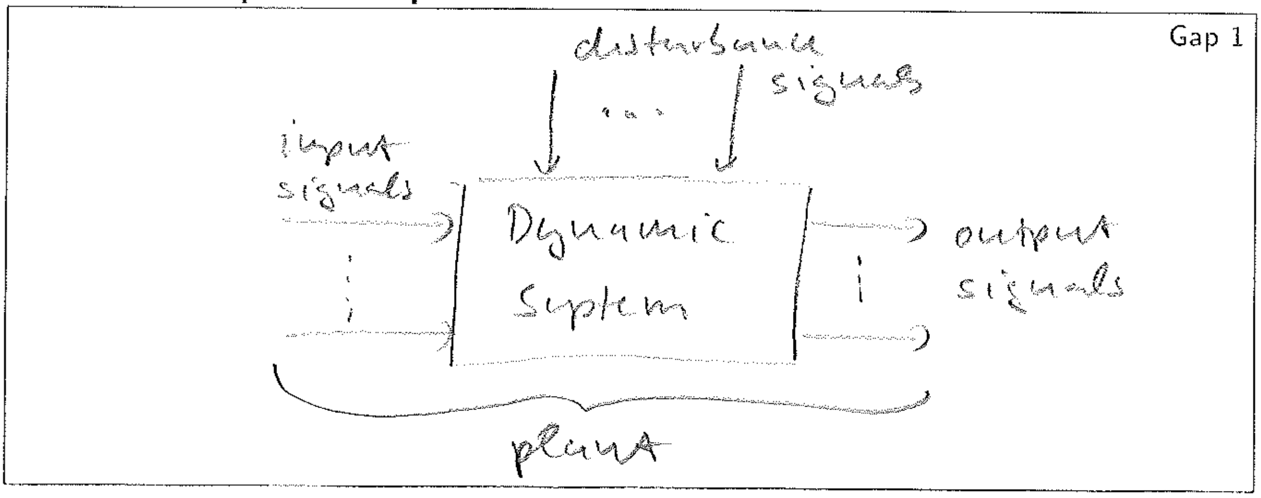
Dynamic System

System that shows dynamic dependency between input and output signals

- Signal is a time-varying physical quantity (e.g. position, velocity, temperature, voltage, current, ...)
- Usually, dynamic systems possess memory
- Dynamic systems are for example modeled by differential equations, difference equations, transfer functions

Motivation: Dynamic System in Control

Detailed Graphical Representation

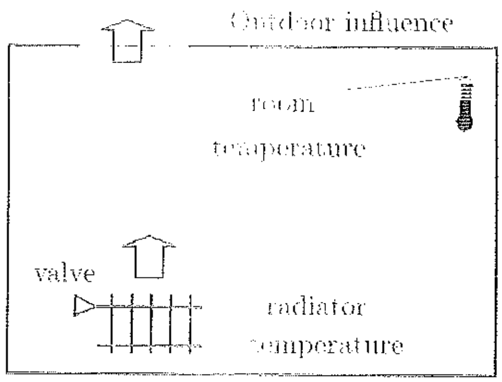


Manipulation of a Dynamic System

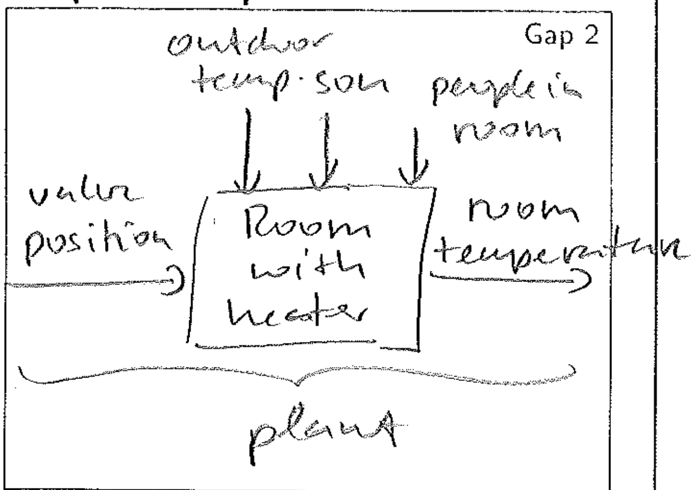
Application of appropriate inputs such that the system state/output behaves as desired even in the presence of disturbances

Examples: Room Temperature System

Schematic



Graphical Representation

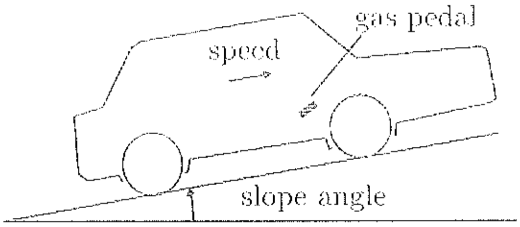


Control Task

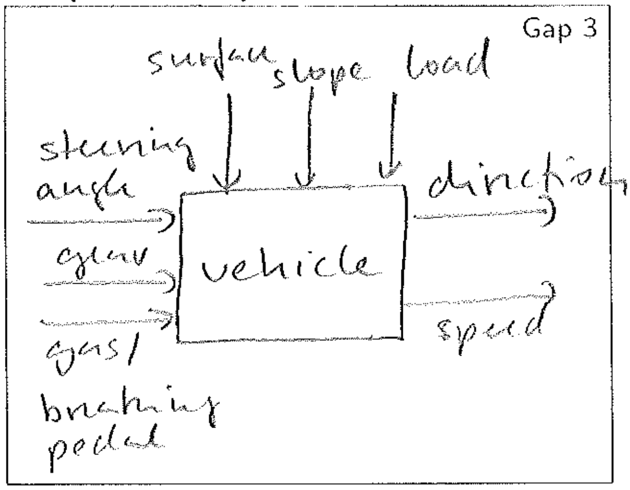
- Desired (specified) behavior: keep room temperature constant
- Manipulation: automatically adjust valve position

Examples: Vehicle Speed

Schematic



Graphical Representation

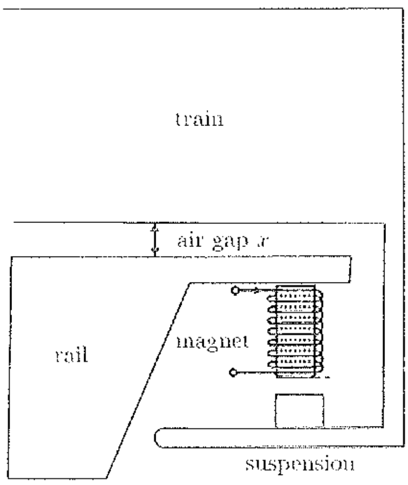


Control Task

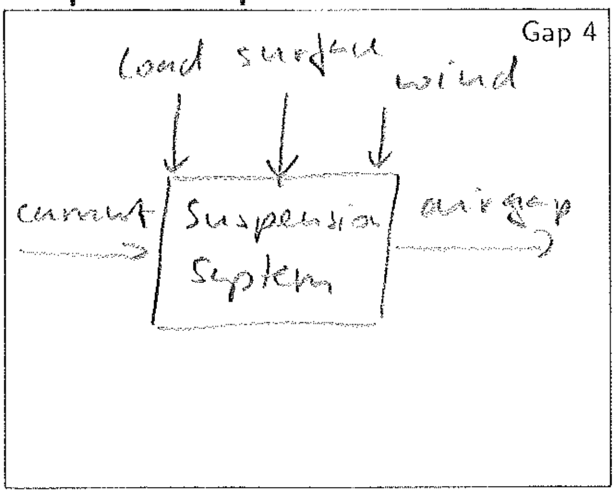
- Desired (specified) behavior: keep vehicle speed constant
- Manipulation: automatically adjust pedal position

Examples: Magnetic Suspension

Schematic



Graphical Representation



Control Task

- Desired (specified) behavior: Move to/keep specified position
- Manipulation: automatically adjust current in the coil

Examples: Movies

Inverted Pendulum

- Desired (specified) behavior: Keep pendulum upright
- Manipulation: horizontally move cart

  ⇒ http://www.rt.eei.uni-erlangen.de/FGnls/video/inverted_pendulum.wmv

Automatic Parking

- Desired (specified) behavior: Reach parking position (without collision)
- Manipulation: automatically adjust speed and steering angle

  ⇒ http://www.rt.eei.uni-erlangen.de/FGnls/video/automatic_parking.wmv

Ball on Plate

- Desired (specified) behavior: follow path/keep position
- Manipulation: automatically change orientation of plate

  ⇒ http://www.rt.eei.uni-erlangen.de/FGnls/video/ball_on_plate.wmv

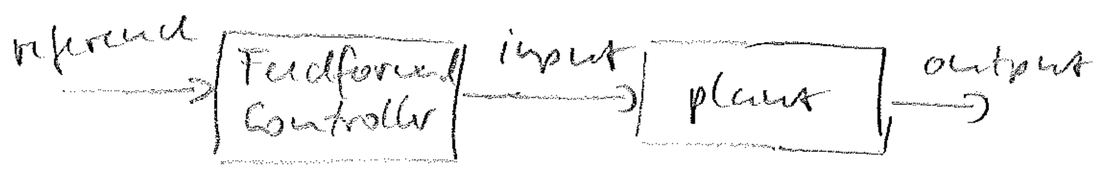
Control Engineering: Basic Task

Main Task of Control Engineering

Design and realize technical appliance – controller – that enforces the desired output behavior of the plant when connected to the plant

Feedforward Control

Gap 5

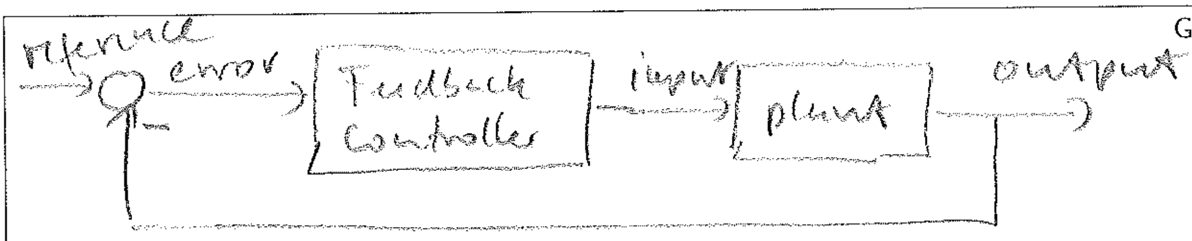


⇒ Feedforward controller provides appropriate input such that the plant follows a given reference signal

Control Engineering: Basic Principles

Feedback Control

Gap 6



⇒ Feedback controller tries to compensate the difference between a reference signal and the measured output signal

Remarks

- Control problems occur in many subject areas
- Examples: process engineering, electrical engineering, communication networks, automotive applications, medicine, chemistry, biology, ...
  ⇒ Mathematical abstraction of control problems to enable interdisciplinary application

Control Engineering: General Solution Procedure

Procedure

1. Mathematical modeling of plant
   ⇒ Abstraction from the physical problem
2. Analysis of the plant behavior
   ⇒ Determine basic properties of the plant and their implications on the design
3. Controller design
   ⇒ Achieve desired plant behavior
4. Simulation and test on the real system
   ⇒ Verify if design goals are achieved

The main subjects of this lecture are items 1 to 3 for systems with a single input signal and a single output signal (SISO)

Plant Modeling: Basic Idea

A plant model is a mathematical description of the cause-effect relationship between the plant signals that are relevant for the design task

Remarks

- Different design tasks for the same system can lead to different plant models
- We focus on models for control tasks
- We consider as relevant signals
  - Input signals (signals that we can directly influence)
  - Output signals (signals that we want to manipulate)
  - Disturbance signals (signals that cannot be directly influenced and that can have a negative effect on the system)

Plant Modeling: Example

RC-Circuit

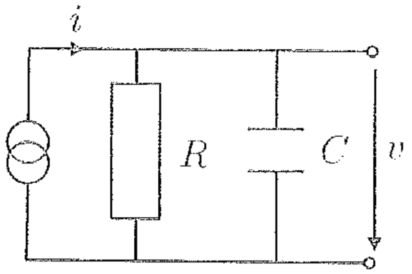
Gap 7

$i = \frac{v}{R} + C \cdot \frac{dv}{dt}$

Use input $u = i$

output $y = v$

$\Rightarrow u = \frac{y}{R} + C \frac{dy}{dt}$



- Input: $u = i$
- Output: $y = v$

Differential Equation

$$C \cdot \frac{dy}{dt} + \frac{1}{R} \cdot y = u$$

Plant Modeling: Example

RLC-Circuit

Gap 8

$i = \frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int v\,dt$

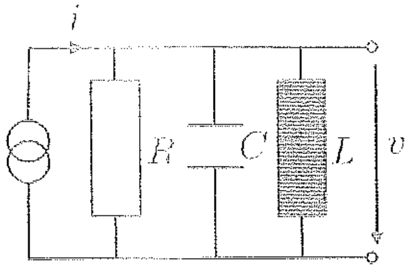
$\Rightarrow$ differentiation

$\frac{di}{dt} = \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{1}{L}\cdot v$

$\Rightarrow$ use input $u = i$

output $y = v$

$\frac{du}{dt} = \frac{1}{R}\frac{dy}{dt} + C\frac{d^2y}{dt^2} + \frac{1}{L}y$



- Input: $u = i$
- Output: $y = v$

Differential Equation

$$C \cdot \frac{d^2y}{dt^2} + \frac{1}{R}\cdot\frac{dy}{dt} + \frac{1}{L}\cdot y = \frac{du}{dt}$$

Plant Modeling: Basic Dynamic Behavior in this Lecture

Linear Time-invariant Ordinary Differential Equation (LTI ODE)

$$a_n y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1\dot{y} + a_0 y = b_m u^{(m)} + b_{m-1}u^{(m-1)} + \cdots + b_1\dot{u} + b_0 u$$

Notation

- y: output signal (in the time domain)
- u: input signal (in the time domain)
- $y^{(i)}$ ($u^{(i)}$): i-th time derivative of y (u)
- a_i, b_i: coefficients in $\mathbb{R}$
- n: highest derivative of y
- m: highest derivative of u

Block Diagram: Description

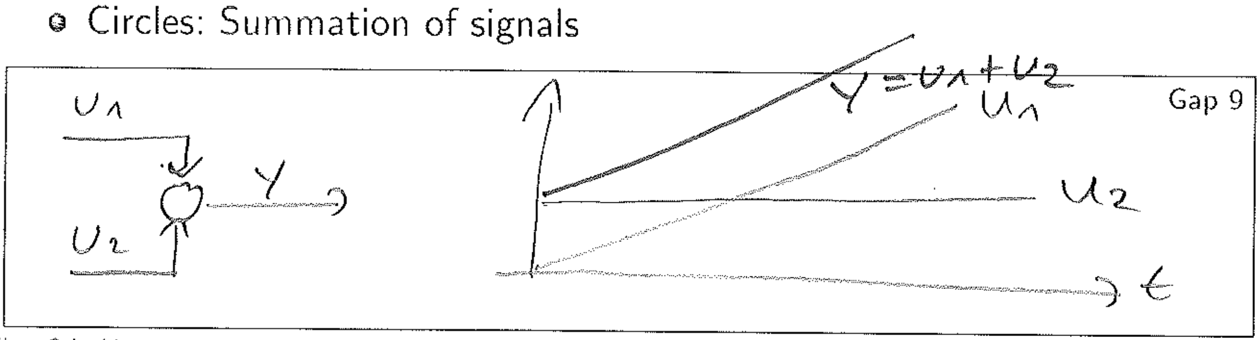
Characteristics of the Block Diagram

A block diagram is a graphical representation of the cause-effect relationship between signals by blocks and directed lines

⇒ Visualization of direction of action and interdependencies

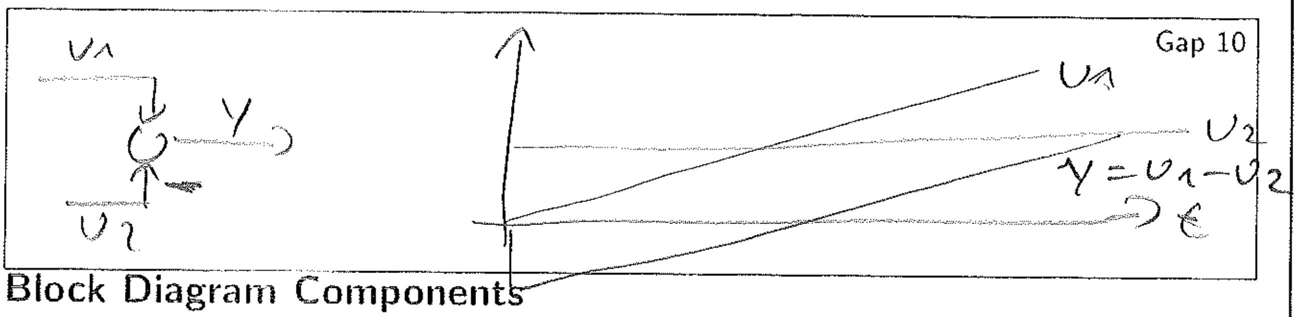
Block Diagram Components

- Directed Lines: System signals and their direction of action
- Circles: Summation of signals



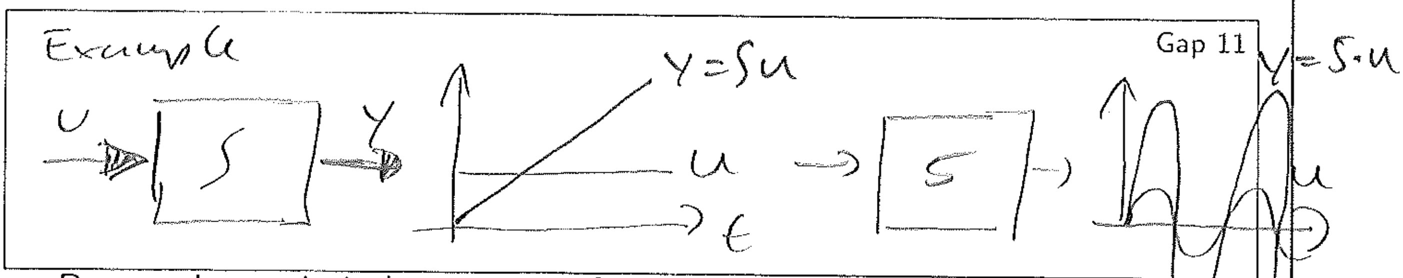
Block Diagram

Block Diagram: Description



Block Diagram Components

- Rectangles: Dynamic relationship between signals: unique mapping from input signal to output signal



⇒ Rectangles and circles are transfer blocks

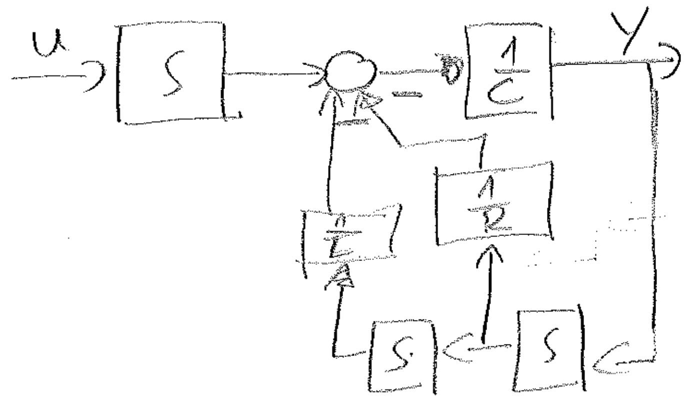
Block Diagram: Examples

RLC-Circuit

Gap 12

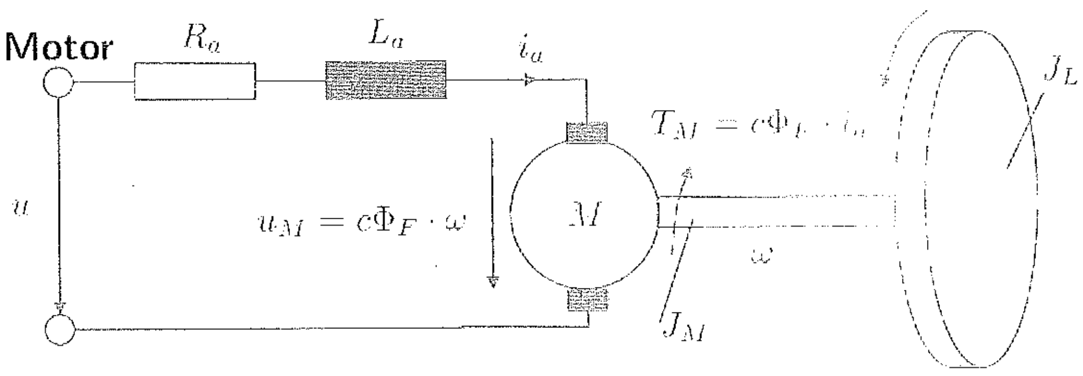
$$\int u\,dt = \frac{1}{R}\int y\,dt + C\cdot y + \frac{1}{L}\int\int y\,dt$$

$$\Rightarrow y = \frac{1}{C}\left(\int u\,dt - \frac{1}{R}\int y\,dt - \frac{1}{L}\int\int y\,dt\right)$$



Plant Modeling: Example

DC-Motor



Variables

- u, i: input voltage, current
- u_M: induced voltage
- ω: rotational velocity
- T_M: motor torque

Parameters

- R_a: resistance
- L_a: inductance
- $c\Phi_F$: motor constant
- J_a: moment of inertia

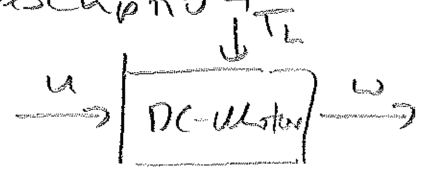
Plant Modeling: System Equations

Computation

Gap 13

Equations

Description



• Electrical circuit

$$u_L = L_a \frac{di_a}{dt} \Rightarrow \frac{di_a}{dt} = \frac{1}{L_a} u_L$$

$$\Rightarrow i_a = \frac{1}{L_a}\int_0^t u_L(\tau)\,d\tau + i_a(0) \quad (1)$$

$$u = R_a i_a + u_L + u_M \Rightarrow u_L = u - R_a i_a - u_M \quad (2)$$

$$u_M = c\Phi_F\, \omega \quad (3)$$

• Mechanical Part: $J\dot{\omega} = T_M - T_L$

$$\Rightarrow \dot{\omega} = \frac{1}{J}(T_M - T_L) \Rightarrow \omega = \frac{1}{J}\int_0^t T_M(\tau) - T_L(\tau)\,d\tau + \omega(0) \quad (4)$$

$$T_M = c\Phi_F \cdot i_a \quad (5)$$

Plant Modeling: Block Diagram

Graphical Representation

Gap 14